Performance of FHSS Systems Employing Carrier Jitter Against One-Dimensional Tone Jamming

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***Abstract*—In this paper, the performance of a frequency-hop spread-spectrum system employing carrier jitter against one-dimensional tone jamming ($n = 1$ band multitone jamming) is investigated. First, noncoherent BFSK signaling under continuous-wave (CW) tone interference with arbitrary frequency offset is analyzed. A closed-form expression is derived for the error probability when there is one interfering CW tone and the background noise is negligible. When the background noise is significant, an expression involving one numerical integration is derived for the probability of error. It is shown that an interfering CW tone with power less than that of the signal can still cause errors with significant probability for certain ranges of carrier offsets. Next, we apply these results in analyzing the performance of a FHSS communications system under one-dimensional tone jamming when the communicator pseudorandomly jitters his carrier frequency from hop to hop. Two different methods of carrier jittering are considered. We find that one of the schemes offers approximately a 3 dB gain in signal-to-noise ratio over a system without carrier jittering while the other scheme offers no significant gain.**

I. Introduction

In this paper, the performance of a frequency-hop spread-spectrum (FHSS) system employing carrier jitter against one-dimensional tone jamming[1] is investigated. In order to achieve this task, we first analyze the error performance of noncoherently demodulated orthogonal Binary Frequency Shift Keying (BFSK) under CW tone interference and additive white Gaussian noise (AWGN.) The case when the frequency of the interfering CW tone falls exactly on that of one of the two BFSK frequencies was investigated in [1]–[5]. Here, a more general case where the interfering CW tone may have an arbitrary frequency offset from the BFSK signals is considered. This may be the case when there is multipath with Doppler shift or when the communicator is under intentional tone jamming with imperfect knowledge of the communicator's band-structure. First, a closed-form expression for the probability of error is derived when there is one interfering CW tone and the background noise is negligible. Numerical evaluation shows that an interfering CW tone with less power than that of the signal can still cause errors with significant probability by selecting a suitable frequency offset. Next, an expression involving one numerical integration is obtained for the probability of error when the background noise is not negligible. Finally, a discrete-time simulation model is derived when there are multiple-interfering CW tones.

These results are applied in analyzing the performance of a FHSS communications system under one-dimensional tone jamming when the frequencies of the jamming tones have arbitrary offsets from those of the BFSK signals used by the communicator. This situation may occur when the jammer does not have an accurate information about the band structure of the communicator or when the communicator intentionally jitters his hopping frequencies pseudorandomly from hop to hop. It was argued loosely in [8] that when this is the case, the effectiveness of the tone jammer is expected to be reduced by approximately 3 dB. We show that this is indeed true when a proper carrier jittering scheme is employed.

The organization of this paper is as follows. First, in Section II, we give a description of the communications system and the channel model being considered. In Section III, we analyze the error performance of noncoherently demodulated (orthogonal) BFSK signaling under CW tone interference with arbitrary frequency offsets. In Section IV, we apply the results of Section III to the analysis of two FHSS systems employing carrier jitter under one-dimensional tone jamming. In Section V, we give numerical results and conclusions are drawn in Section VI.

II. System and Channel Model

The communications system considered in this Section and the following is a standard narrowband communications system employing BFSK with noncoherent demodulation. The communications channel is degraded by AWGN and a set of CW tones with arbitrary frequencies. A sequence of equally likely binary symbols $b_i \in \{+1, -1\}$ from the source is BFSK modulated and transmitted over the waveform channel. The complex baseband equivalent of the transmitted signal $s(t)$ during the transmission of symbol b_0 in time interval $[0, T)$ is given as follows,

$$s(t) = \sqrt{2S}\exp(j2\pi b_0 \Delta f t + j\phi_s) \tag{1}$$

where $\frac{1}{T}$ denotes the bit rate, $j = \sqrt{-1}$, S denotes the signal power, $2\Delta f = \frac{1}{T}$ is the frequency separation between the two BFSK frequencies and ϕ_s is a random phase introduced by the modulator which is assumed to be uniformly distributed

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[1] Also referred to as $n = 1$ band multitone jamming [10].

on $[0, 2\pi)$. The complex lowpass equivalent signal seen by the receiver during the time interval $[0, T)$ is then,

$$r(t) = s(t) + \sum_{i=1}^{N} J_i(t) + n(t) \tag{2}$$

where $n(t)$ is a complex lowpass AWGN process with double-sided power spectral density N_0 and $J_i(t)$ is the complex lowpass equivalent of the ith interfering CW tone given by

$$J_i(t) = \sqrt{\frac{2S}{\alpha_i}} \exp(j2\pi\delta f_i t + j\phi_i). \tag{3}$$

Here, $\alpha_i > 0$ denotes the ratio of power between the communicator's signal and the ith interfering CW tone, δf_i is an arbitrary frequency offset and ϕ_i's are independent random phase terms assumed to be uniformly distributed on $[0, 2\pi)$.

The receiver is the standard noncoherent demodulator optimal under AWGN [6], [10]. To decide which symbol was transmitted in the interval $[0, T)$, the receiver first computes two decision variables $|U_1|$ and $|U_{-1}|$ where

$$U_l = \frac{1}{\sqrt{2ST}} \int_0^T r(t) \exp(-j2\pi l \Delta f t)\, dt \quad l = -1, +1. \tag{4}$$

The receiver decides that $b_0 = +1$ was transmitted if $|U_{+1}| > |U_{-1}|$ and vice versa. Using (2), (3), and (4), it can be shown that U_1 and U_{-1} can be written as follows given that $b_0 = +1$ was transmitted:

$$U_1 = e^{j\phi_s} + \sum_{i=1}^{N} \frac{1}{\sqrt{\alpha_i}} \mathrm{sinc}\left(\mu_i - \frac{1}{2}\right) e^{j\phi_i + j\pi(\mu_i - \frac{1}{2})} + z_1 \tag{5}$$

$$U_{-1} = \sum_{i=1}^{N} \frac{1}{\sqrt{\alpha_i}} \mathrm{sinc}\left(\mu_i + \frac{1}{2}\right) e^{j\phi_i + j\pi(\mu_i + \frac{1}{2})} + z_{-1} \tag{6}$$

where $\mu_i = \delta f_i T$ is the normalized frequency offset of the ith CW tone and $\mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$. Also, z_i's are independent complex Gaussian random variables with zero mean and variance equal to $1/(\frac{2E_b}{N_0})$ where $E_b = ST$ is the energy per transmitted binary symbol.

III. Probability of Error

A. One CW Tone Interferer and No Background Noise

We begin the analysis of the probability of error with the simple case when there is one interfering CW tone ($N = 1$) and the background thermal noise is negligible. For this case, the decision variables are given as

$$|U_1| = \left| e^{j\phi_s} + \frac{1}{\sqrt{\alpha}} \mathrm{sinc}\left(\mu - \frac{1}{2}\right) e^{j\phi_1 + j\pi(\mu - \frac{1}{2})} \right| \tag{7}$$

$$|U_{-1}| = \left| \frac{1}{\sqrt{\alpha}} \mathrm{sinc}\left(\mu + \frac{1}{2}\right) e^{j\phi_1 + j\pi(\mu + \frac{1}{2})} \right|. \tag{8}$$

Note that since $|U_i| = |U_i e^{j\phi}|$, multiplying (7) and (8) by appropriate phase terms, we may write the two decision variables as

$$|U_1| = \left| e^{j\phi} + \frac{1}{\sqrt{\alpha}} \mathrm{sinc}\left(\mu - \frac{1}{2}\right) \right| \tag{9}$$

$$|U_{-1}| = \left| \frac{1}{\sqrt{\alpha}} \mathrm{sinc}\left(\mu + \frac{1}{2}\right) \right| \tag{10}$$

where $\phi = (\phi_s - \phi_1 - \pi(\mu - \frac{1}{2}))$ considered modulo 2π is a random variable uniformly distributed on $[0, 2\pi)$. Hence, the probability of error as a function of μ and α is given by

$$\begin{aligned} P_{e,1}(\mu, \alpha) &= \Pr\{|U_1| < |U_{-1}| | b_0 = +1\} & (11)\\ &= \Pr\{|U_1|^2 < |U_{-1}|^2 | b_0 = +1\} & (12)\\ &= \Pr\left\{ \mathrm{sinc}\left(\mu - \frac{1}{2}\right) \cos(\phi) < h(\alpha, \mu) \right\} & (13) \end{aligned}$$

where

$$h(\alpha, \mu) = \frac{1}{2\sqrt{\alpha}} \left[\mathrm{sinc}^2\left(\mu + \frac{1}{2}\right) - \mathrm{sinc}^2\left(\mu - \frac{1}{2}\right) - \alpha \right]. \tag{14}$$

For the special case when $\mu = \frac{1}{2}$, i.e., the frequency of the jamming signal coincides with that of the desired signal, (13) reduces to

$$P_{e,1}(\frac{1}{2}, \alpha) = \Pr\left\{ \cos(\phi) < -\frac{1}{2\sqrt{\alpha}}(1 + \alpha) \right\} = 0, \quad \forall \alpha > 0 \tag{15}$$

and when $\mu = -\frac{1}{2}$, i.e., the frequency of the jamming signal coincides with the BFSK frequency corresponding to $b_0 = -1$, (13) reduces to

$$P_{e,1}(-\frac{1}{2}, \alpha) = \Pr\{0 < (1 - \alpha)\} \tag{16}$$

which is equal to 1 when $\alpha < 1$ (interfering signal has larger power than the intended signal) and 0 when $\alpha > 1$ as expected. Otherwise, calculation shows that (13) can be written as

$$\begin{aligned} P_{e,1}(\mu, \alpha) = {} & P'(\mu, \alpha) \cdot I\left(\mathrm{sinc}\left(\mu - \frac{1}{2}\right) > 0 \right) \\ & + (1 - P'(\mu, \alpha)) \cdot I\left(\mathrm{sinc}\left(\mu - \frac{1}{2}\right) < 0 \right) \end{aligned} \tag{17}$$

where

$$P'(\mu, \alpha) = \begin{cases} \frac{1}{\pi} \cos^{-1}(g(\mu, \alpha)) & |g(\mu, \alpha)| \le 1 \\ 0 & g(\mu, \alpha) > 1 \\ 1 & g(\mu, \alpha) < -1 \end{cases} \tag{18}$$

and

$$g(\mu, \alpha) = \frac{\alpha - \mathrm{sinc}^2(\mu + \frac{1}{2}) + \mathrm{sinc}^2(\mu - \frac{1}{2})}{2\sqrt{\alpha}\,\mathrm{sinc}(\mu - \frac{1}{2})}. \tag{19}$$

Here, $I(\mathrm{C}) = 1$ if the condition C is true and 0 otherwise. By symmetry, the probability of error when $b_0 = -1$ is transmitted is given by $P_{e,-1}(\mu, \alpha) = P_{e,+1}(-\mu, \alpha)$. Hence, the average probability of error is given by

$$P_e(\mu, \alpha) = \frac{1}{2}[P_{e,+1}(\mu, \alpha) + P_{e,+1}(-\mu, \alpha)]. \tag{20}$$

For the case when μ is a random variable, we may simply integrate (20) over the distribution of μ to find the average probability of error as

$$P_e(\alpha) = \int_{-\infty}^{\infty} P_e(\mu, \alpha) p_\mu(\mu)\, d\mu \tag{21}$$

where $p_\mu(\cdot)$ is the probability density function of the normalized frequency offset μ.

B. One CW Tone Interferer and Background Noise

Now we consider the case when there is one interfering CW tone and the background noise is not negligible. In this case, the two decision variables are given as

$$|U_1| = \left| e^{j\phi_s} + \frac{1}{\sqrt{\alpha}} \mathrm{sinc}\left(\mu - \frac{1}{2}\right) e^{j\phi_1 + j\pi(\mu - \frac{1}{2})} + z_1 \right| \tag{22}$$

$$|U_{-1}| = \left| \frac{1}{\sqrt{\alpha}} \mathrm{sinc}\left(\mu + \frac{1}{2}\right) e^{j\phi_1 + j\pi(\mu + \frac{1}{2})} + z_{-1} \right|. \tag{23}$$

Multiplying by appropriate phase terms, we have

$$|U_1| = \left| e^{j\phi} + \frac{1}{\sqrt{\alpha}} \mathrm{sinc}\left(\mu - \frac{1}{2}\right) + z_1 e^{-j(\phi_1 + a)} \right| \tag{24}$$

$$|U_{-1}| = \left| \frac{1}{\sqrt{\alpha}} \mathrm{sinc}\left(\mu + \frac{1}{2}\right) + z_{-1} e^{-j(\phi_1 + b)} \right| \tag{25}$$

where $\phi = (\phi_s - \phi_1 - a)$, $a = \pi(\mu - \frac{1}{2})$ and $b = \pi(\mu + \frac{1}{2})$. Since z_i's are spherically symmetric random variables, we may replace $z_1 e^{-j(\phi_1 + a)}$ and $z_{-1} e^{-j(\phi_1 + b)}$ by $\hat{z}_1$ and $\hat{z}_{-1}$ where the statistics of $\hat{z}_1$, $\hat{z}_{-1}$ are identical to those of z_1 and z_{-1}. Hence, conditioned on μ, we may write,

$$|U_1| = \left| e^{j\phi} + \frac{1}{\sqrt{\alpha}} \mathrm{sinc}\left(\mu - \frac{1}{2}\right) + \hat{z}_1 \right| \tag{26}$$

$$|U_{-1}| = \left| \frac{1}{\sqrt{\alpha}} \mathrm{sinc}\left(\mu + \frac{1}{2}\right) + \hat{z}_{-1} \right| \tag{27}$$

and ϕ is a random variable uniformly distributed on $[0, 2\pi)$ independent of $\hat{z}_i$.

Note that U_{+1} and U_{-1} are sums of independent spherically symmetric random variables, and thus they are themselves spherically symmetric [7]. Now, we may use the following known result for spherically symmetric random variables to compute the probability of error [1, 6].

Theorem 1: Let X, Y be independent spherically symmetric random variables. Then

$$\Pr(|X| < |Y|) = -\int_0^{\infty} \Phi_X(s) \frac{d\Phi_Y(s)}{ds}\, ds \tag{28}$$

where $\Phi_X(s)$ and $\Phi_Y(s)$ are the characteristic functions of X and Y, respectively.

The characteristic functions of U_{+1} and U_{-1} can be computed to be

$$\Phi_{+1}(s) = e^{-\frac{s^2}{4\gamma}} J_0(s) J_0(s f_m(\mu, \alpha)) \tag{29}$$

$$\Phi_{-1}(s) = e^{-\frac{s^2}{4\gamma}} J_0(s f_p(\mu, \alpha)) \tag{30}$$

where $\gamma = \frac{E_b}{N_0}$ denotes the signal-to-noise ratio, $f_p(\mu, \alpha) = \frac{1}{\sqrt{\alpha}} \mathrm{sinc}(\mu + \frac{1}{2})$ and $f_m(\mu, \alpha) = \frac{1}{\sqrt{\alpha}} \mathrm{sinc}(\mu - \frac{1}{2})$. Also $J_0(\cdot)$ is the Bessel function of the first kind of zeroth order defined as $J_0(z) = \frac{1}{\pi} \int_0^{\pi} \cos(z \sin(\theta)) d\theta$ [9]. Applying Theorem 1 and after some algebra, the probability of error conditioned on μ and α may be derived as

$$P_e(\mu, \alpha) = \int_0^{\infty} \Phi(s, \mu, \alpha)\, ds \tag{31}$$

where

$$\Phi(s, \mu, \alpha) = e^{-\frac{s^2}{2\gamma}} J_0(s) J_0(s f_m(\mu, \alpha)) \times \left[\frac{s}{2\gamma} J_0(s f_p(\mu, \alpha)) + f_p(\mu, \alpha) J_1(s f_p(\mu, \alpha)) \right]. \tag{32}$$

An alternate form for the conditional probability of error $P_e(\mu, \alpha)$ may be derived as follows using the fact that $|U_1|$, $|U_{-1}|$ given by (26)–(27) conditioned on ϕ are Rician distributed.

$$P_e(\mu, \alpha) = \int_0^{2\pi} P_e(\mu, \alpha, \phi) d\phi \tag{33}$$

where

$$\begin{aligned} P_e(\mu, \alpha, \phi) &= Q\left(\sqrt{\gamma f_p^2(\mu, \alpha)}, \sqrt{\gamma(1 + f_m^2(\mu, \alpha) + 2 f_m(\mu, \alpha) \cos(\phi))}\right) \\ &\quad + \frac{1}{2} \exp\left(\frac{\gamma}{2}[1 + f_m^2(\mu, \alpha) + f_p^2(\mu, \alpha) + 2 f_m(\mu, \alpha) \cos(\phi)]\right) \\ &\quad \times I_0\left(\gamma f_p(\mu, \alpha) \sqrt{1 + f_m^2(\mu, \alpha) + 2 f_m(\mu, \alpha) \cos(\phi)}\right) \end{aligned} \tag{34}$$

and $Q(\cdot, \cdot)$, $I_0(\cdot)$ are respectively the Marcum-Q and the modified Bessel function of the zeroth order [9].

C. Multiple CW Tone Interferer and Background Noise

In this case, the decision variables are the magnitudes of U_1 and U_{-1} given by (5) and (6). Here, $|U_1|$ and $|U_{-1}|$ are not independent and hence Theorem 1 does not apply. The probability of error for this case may be estimated by performing discrete-time Monte Carlo simulations using (5) and (6).

IV. Applications to FHSS Communications System Under One-Dimensional Tone Jamming

Here, we apply the results derived in the previous sections to the analysis of a FHSS communications system under one-dimensional tone jamming. Specifically, we consider the case when the jammer is not aware of the exact band-structure of the communicator. That is, the jammer does not know the exact frequencies of the BFSK signals being used by the communicator in the spread-spectrum bandwidth. Hence, the one-dimensional tone jammer which places at most one interfering CW tone per slot is not able to place its CW tones precisely

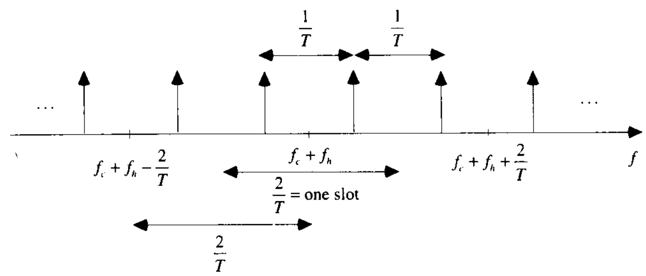

Fig. 1. Location of BFSK signal frequencies for the FHSS system.

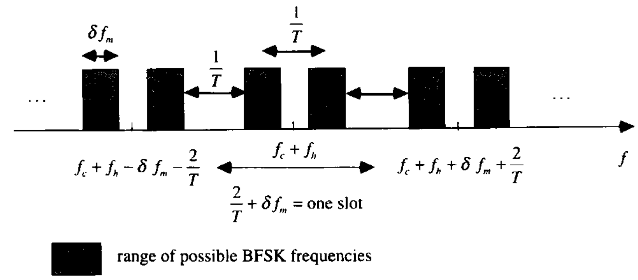

Fig. 2. Location of the BFSK signal frequencies for the FHSS system using carrier jittering Scheme I.

on the frequencies of one of the BFSK signals used by the communicator as is the usual assumption when analyzing the performance of FHSS systems under tone-jamming [10]. This situation may occur when the jammer does not have enough knowledge about the communicator's system (optimistic from the communicator's viewpoint) or when the communicator intentionally jitters its carrier frequency in a pseudorandom fashion from hop to hop deterring the jammer from finding out the exact frequency location of the transmitted signal.

We consider a FHSS system [10] using a bandwidth of W_{SS} Hz operating over a channel dominated by a one-dimensional tone jammer so that the background noise may be neglected. One binary symbol is transmitted per hop using orthogonal BFSK with signal power S. The transmitted signal during a hop can be written as follows:

$$s(t)=\sqrt{2S}\cos\left(2\pi\left(f_c+f_h+\frac{b_i}{2T}\right)t+\phi_s\right) \tag{35}$$

where f_c is the main carrier frequency, $f_h\in\left\{0,\frac{2}{T},\frac{4}{T},\ldots,\frac{2(q-1)}{T}\right\}$ is the hop frequency offset with respect to f_c and $\frac{b_i}{2T}$ is the frequency offset from f_c+f_h due to BFSK modulation of data symbol b_i. The locations of the BFSK frequencies are depicted in Fig. 1. The jammer has power J which is divided into Q CW tones. The jammer distributes these tones over the bandwidth W_{SS} such that there is at most one CW tone per frequency slot[2].

We consider two different schemes for carrier jittering. For both schemes, it is assumed that the jammer places its CW tones at $\pm\frac{1}{2T}$ from f_c+f_h. In the first scheme (Scheme I,) an additional δf_m Hz of bandwidth is allocated to each slot and the hop frequency is given a uniform distribution on $\left[-\frac{\delta f_m}{2},+\frac{\delta f_m}{2}\right]$ from the center of the slot. The transmitted signal during a hop in this case is given as follows:

$$s(t)=\sqrt{2S}\cos\left(2\pi\left(f_c+f_h+\delta f+\frac{b_i}{2T}\right)t+\phi_s\right) \tag{36}$$

where $f_h\in\left\{0,\frac{2}{T}+2\delta f_m,\frac{4}{T}+(q-1)\,\delta f_m,\ldots,\frac{2(q-1)}{T}+\delta f_m\right\}$ and δf is a random variable uniformly distributed on $\left[-\frac{\delta f_m}{2},+\frac{\delta f_m}{2}\right]$. The range of possible BFSK frequencies for this scheme is shown in Fig. 2. For a fixed available bandwidth, this results in a smaller number of slots available for the communicator. Hence, for this scheme, the jammer requires larger power for each of its CW tones to compensate for the frequency offset but requires a smaller number of them because of the bandwidth expansion of the communicator due to jittering. The second scheme (Scheme II) is identical to the first scheme except that additional δf_m Hz of bandwidth is not allocated to the slots. For this case, bandwidth expansion due to carrier jittering is negligible but the possibly adverse effects from jammers' CW tones falling in the neighboring slots must be accounted for. Performances of the two FHSS systems employing carrier jitter under one-dimensional tone jamming are evaluated in the following subsections.

[2] A frequency slot is defined as the $\frac{2}{T}$ Hz of bandwidth centered about a given hop frequency f_c+f_h.

A. Scheme I

An equivalent model of this system for a particular jammed hop is a BFSK modulated signal jammed by a CW tone whose normalized frequency offset denoted by μ is a random variable with a uniform distribution on $A\triangleq\left[\frac{1}{2}(1-\mu_m),\frac{1}{2}(1+\mu_m)\right]$ with probability $\frac{1}{2}$ and $B\triangleq\left[\frac{1}{2}(-1-\mu_m),\frac{1}{2}(-1+\mu_m)\right]$ with probability $\frac{1}{2}$ where $\mu_m=\delta f_m T$. Due to the extra δf_m Hz of bandwidth allocated to each slot for this scheme, we may safely ignore the effect of jammers' CW tones landing on the neighboring slots.

Without frequency jittering, the probability of a hop being hit by the jammer for the system under consideration can be shown to be [10]

$$P_h=\min\left\{1,\frac{2\alpha}{\gamma_J}\right\} \tag{37}$$

where $\alpha=\frac{S}{J/Q}$, $\gamma_J=\frac{E_b}{N_J}$ and $N_J=\frac{J}{W_{\mathrm{SS}}}$. When the communicator jitters its hopping frequencies as described above, taking into account the bandwidth expansion due to the additional δf_m Hz of bandwidth allocated for each slot, the probability of hit is given as

$$P_{h,\mathrm{jitter}}=(1+\mu_m)P_h. \tag{38}$$

The jammer's goal is to choose the factor α to maximize the probability of error. Without loss of generality, we assume that $b_i=+1$. Then the average probability of error for the system is given by the following equation:

$$P_1(\mu_m)=\max_{0<\alpha<\frac{\gamma_J}{2(1+\mu_m)}}\left[\frac{2\alpha(1+\mu_m)}{\gamma_J}P_a(\mu_m,\alpha)\right] \tag{39}$$

where

$$P_a(\mu_m,\alpha)=\frac{1}{2\mu_m}\left[\int_A P_{e,1}(\mu,\alpha)\,d\mu+\int_B P_{e,1}(\mu,\alpha)\,d\mu\right] \tag{40}$$

and $P_{e,1}(\mu,\alpha)$ is given by (17).

Numerical results given in Section V show that the decrease in the number of slots due to the bandwidth expansion of the jittering cancels the gain achieved from the randomness introduced by jittering and this scheme offers negligible gain compared to a system without carrier jittering.

B. Scheme II

For this carrier jittering scheme, we need to consider the effects of the jammers' CW tones landing on the neighboring slots as well as the one corresponding to the slot being used by the communicator at the time. In order to keep the computation time at a manageable level, we assume that the normalized hop frequency offsets may take one of M discrete frequencies uniformly spaced in $\left[-\frac{\mu_m}{2}, +\frac{\mu_m}{2}\right]$ with equal probability.

Let us consider a particular hop and denote the normalized hop frequency offset for this hop as μ. We assume that $\frac{\mu_m}{2}$ is on the order of $\frac{1}{2}$ and take into account the following normalized frequencies of the jammers' CW tones

$$A_1 = \{-(2.5+\mu), -(1.5+\mu)\} \tag{41}$$

$$A_2 = \{-(0.5+\mu), 0.5-\mu\} \tag{42}$$

$$A_3 = \{1.5-\mu, 2.5-\mu\}. \tag{43}$$

The sets A_1 and A_3 corresponds to the normalized locations of the jammers' CW tone frequencies in the frequency slots immediately to the left and right of that used by the communicator and A_2 correspond to those located in the slot corresponding to the one used by the communicator. Then, assuming that the hits to the three slots under consideration are independent, the probability of error can be written as

$$P_1 = \max_{0<\alpha<\frac{2J}{2}} P_a(\alpha) \tag{44}$$

where

$$\begin{aligned} P_a(\alpha) = \sum_{\mu} p_\mu(\mu) \Bigg\{ & \sum_{a \in \{A_1 \cup A_2 \cup A_3\}} P_1(a,\alpha) \cdot \frac{1}{2} P_h (1-P_h)^2 \\ & + \Bigg[\sum_{a_1 \in A_1} \sum_{a_2 \in A_2} P_2((a_1,a_2),\alpha) \\ & + \sum_{a_1 \in A_1} \sum_{a_3 \in A_3} P_2((a_1,a_3),\alpha) \\ & + \sum_{a_2 \in A_2} \sum_{a_3 \in A_3} P_2((a_2,a_3),\alpha) \Bigg] \cdot \frac{1}{4} \cdot P_h^2 (1-P_h) \\ & + \sum_{a_1 \in A_1} \sum_{a_2 \in A_2} \sum_{a_3 \in A_3} P_3((a_1,a_2,a_3),\alpha) \cdot \frac{1}{8} \cdot P_h^3 \Bigg\}. \end{aligned} \tag{45}$$

Here, $P_1(a_i,\alpha)$ denotes the conditional error probability given that one CW jamming tone is present in A_i and $P_2((a_i,a_j),\alpha)$ and $P_3((a_i,a_j,a_k),\alpha)$ are the probability of error given that there are two and three interfering CW tones are present in $\{A_i, A_j\}$ and $\{A_i, A_j, A_k\}$, respectively. The value for $P_1(a_i,\alpha)$ is computed using (17) and those of $P_2((a_i,a_j),\alpha)$ and $P_3((a_i,a_j,a_k),\alpha)$ are estimated through Monte Carlo simulations using (5) and (6) for the numerical results. Also, $p_\mu(\mu) = \frac{1}{M}$ denotes the probability mass function of the discrete random variable μ. We note that as $\frac{E_b}{N_J}$ increases (as P_h decreases,) the contribution of the triple summation term in (45) becomes smaller due to the P_h^3 term. Similar argument holds for the double summation terms (terms involving the P_h^2 term) for larger $\frac{E_b}{N_J}$, and the single summation term which can be analytically computed from (17) will be the only significant term for this case.

If we assume that the only significant term is that due to a jammer having a single tone falling within the slot being jammed (but offset from either of the tones being transmitted), then it is possible to show that for large $\frac{E_b}{N_J}$, the error probability can be expressed in the following form for the worst case jammer when the normalized carrier is uniformly distributed between $-\mu_m$ and $+\mu_m$.

$$P_1 = \frac{K_1}{E_b/N_J}, \quad E_b/N_J \geq K_2 \tag{46}$$

and the optimal jamming fraction P_h

$$P_h = \frac{K_2}{E_b/N_J}, \quad E_b/N_J \geq K_2. \tag{47}$$

The numerical values for K_1 and K_2 depend on the maximum jittering level μ_m. For $\mu_m = 0.5$, the values for K_1 and K_2 are $K_1 = 0.488$ and $K_2 = 1.49$, respectively.

V. Numerical Results

Numerical results are presented in this section. We begin with the error performance of orthogonal BFSK signaling under CW tone interference. First, we consider the case when the background noise is negligible and there is one interfering CW tone. The probability of error as a function of the normalized frequency offset $\mu = \delta f T$ for various values of α are shown in Fig. 3. We see that when $\alpha < 1$ (jammer has larger power than the communicator,) there is a region around $\mu = \pm\frac{1}{2}$ where the probability of error is $\frac{1}{2}$. In this region, the interferer is able to inject enough energy into the wrong correlator forcing the demodulator to make erroneous decisions even though the frequency of the interfering signal has a nonzero frequency offset from the frequencies of the BFSK signals. When $\alpha > 1$ (jammer has less power than the communicator), there are regions of μ where the probability of error is nonzero even though the CW tone is not able to cause any errors when the frequency offset μ is either $+\frac{1}{2}$ or $-\frac{1}{2}$. This is because when there is a nonzero frequency offset, the interfering CW tone affects both of the correlator outputs corresponding to $b = +1$ and -1. The maximum error probability in this case is obtained when $\mu = 0$, i.e., when the jamming signal is placed at the midpoint between the BFSK frequencies. As α increases beyond a certain point, e.g., $\alpha > 1.62$ the probability of error becomes zero for all values of μ. Hence, as with the case when the interfering signal is placed at either of the BFSK frequencies, there is a minimum power that the interfering signal must have in order to cause errors with a nonzero probability. In Fig. 4, the probability of error is plotted for the case when $\alpha = 0.01$, i.e., the interfering signal has 20 dB more power than the communicator. We observe

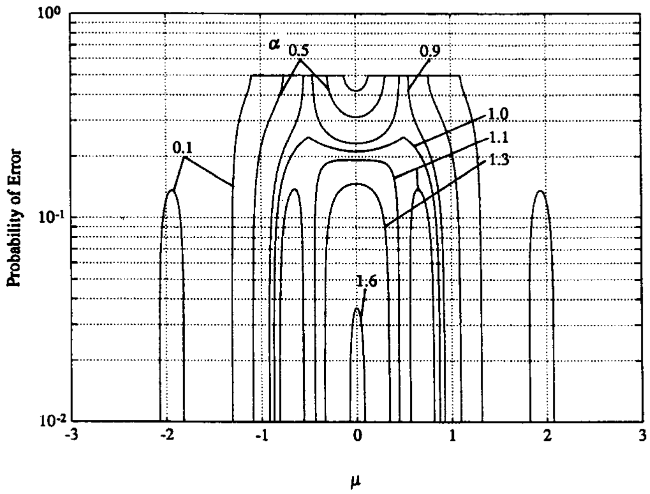

Fig. 3. Probability of error for one interfering CW tone and no background noise for $\alpha = 0.1, 0.5, 0.9, 1.0, 1.1, 1.3, 1.6$.

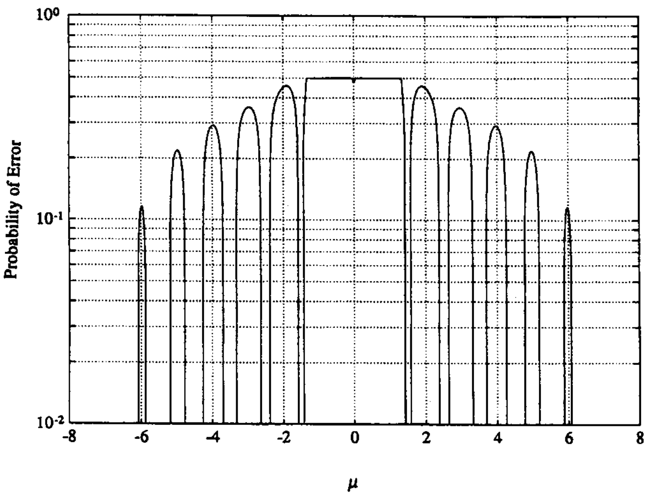

Fig. 4. Probability of error for one interfering CW tone and no background noise for $\alpha = 0.01$.

that the μ axis is divided into regions where the probability of error is nonzero and regions where the probability of error is nonzero. This is due to the fact that for values of μ near $n - \frac{1}{2}$ when $b = +1$ (or $n + \frac{1}{2}$ when $b = -1$,) for $n = \pm 1, \pm 2, \pm 3 \ldots$, the contribution of the interfering signal to the output of the wrong correlator is very small [see (10)]. Next in Fig. 5, we plot the average probability of error for the case when μ has a uniform distribution on $[-\mu_m, +\mu_m]$ for $\alpha = 0.1, 0.5, 1.0$ and 1.5 as a function of μ_m. We note that there exits a nonzero μ_m that maximizes the probability of error for small values of α as may be expected from Fig. 3.

In Fig. 6, the probability of error is plotted for various values of α when the background noise is present and the signal-to-noise ratio $\frac{E_b}{N_0}$ is 10 dB. We note that the outer sidelobes which were present only for very small values of α when the background noise was neglected are present for all values of α though they decrease rather quickly as α and μ increase. Moreover, the jamming tone can cause significant degradation even for values of α that cause no errors when background noise is negligible.

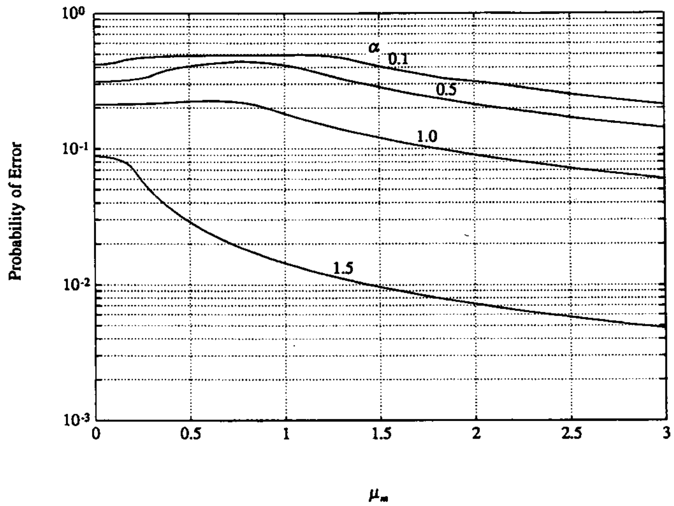

Fig. 5. Probability of error for one interfering CW tone and no background noise when the frequency offset is uniformly distributed on $[-\mu_m, \mu_m]$.

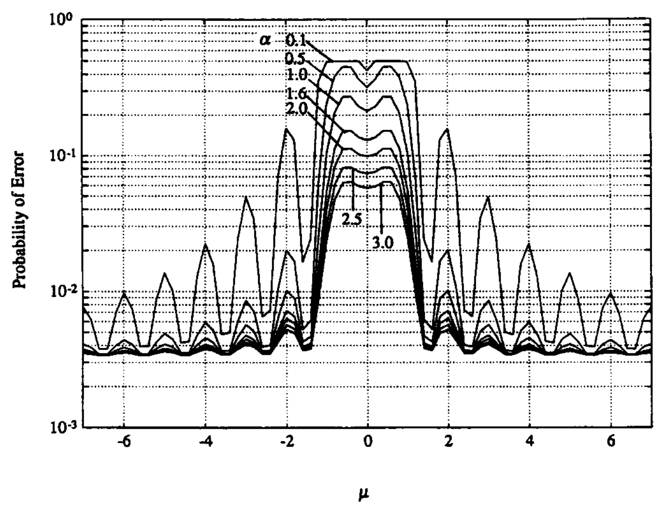

Fig. 6. Probability of error for one interfering CW tone with background noise for $\alpha = 0.1, 0.5, 1.0, 1.6, 2.0, 2.5, 3.0$, and $E_b/N_0 = 10$ dB.

Now we consider the error performance of FHSS systems with the two types of carrier jittering scheme described in Section IV. First in Fig. 7, we show the uncoded error probability for a FHSS system employing Scheme I carrier jittering against a one-dimensional tone jammer versus $\frac{E_b}{N_J}$ computed from (39) with μ_m as the parameter. The case when $\mu_m = 0.0$ denotes the case when there is no carrier jittering. We note that the gain achieved by randomizing the hopping frequencies is negligible (less than $\frac{1}{4}$ dB) for this type of jittering scheme. This is explained by the fact that the decrease in the probability of error when a hop is hit is countered by an increase in the probability of hit by the jammer. Next, the performance of a FHSS system using Scheme II carrier jittering is considerd. It is assumed that μ is equally likely to be any one of the values in $\{-0.5, -0.4, \cdots, 0.4, 0.5\}$. The error performance is computed using (45) or lower bounds to (45)[3] as described

[3]Note that (45) itself is a lower bound since it only takes into account the effects of hits from the slots adjacent to the one being used.

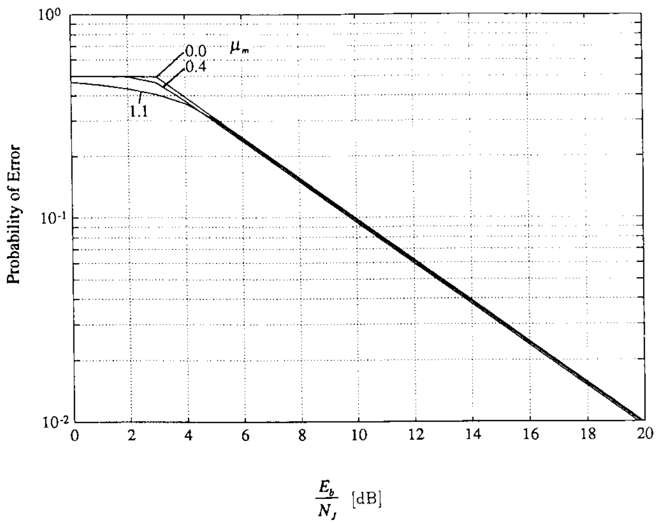

Fig. 7. Uncoded error probability for a FHSS system with Scheme I carrier jittering under one-dimensional tone jamming.

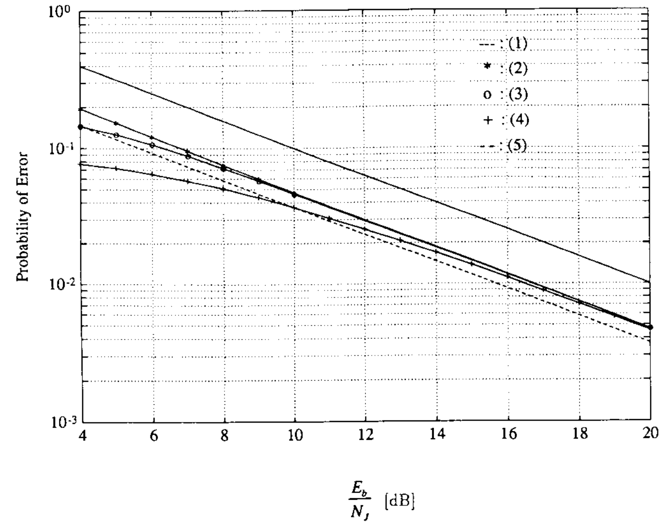

Fig. 8. Uncoded error probability for a FHSS system with Scheme II carrier jittering under one-dimensional tone jamming.

below. The error performance curves for the following cases are plotted in Fig. 8.

1) FHSS system without carrier jittering.
2) Scheme II carrier jittering using (45).
3) Scheme II carrier jittering using (45) but ignoring the triple summation term involving P_h^3. This provides a lower bound for case 2.
4) Scheme II carrier jittering using (45) but ignoring the triple and double summation terms involving P_h^2 and P_h^3. This provides a lower bound for case 3 and thus for case 2.
5) FHSS system under worst case partial-band noise jamming [10] with worst case symbol error probability P_{PBNJ} given by

$$P_{\text{PBNJ}} = \begin{cases} \frac{1}{2} e^{-\frac{E_b}{2N_0}}, & \frac{E_b}{N_0} \leq 3.01 \text{ dB} \\ \frac{e^{-1}}{E_b/N_0}, & \frac{E_b}{N_0} > 3.01 \text{ dB}. \end{cases} \tag{48}$$

We note that both case 3 and case 4 turn out to be very tight lower bounds to the actual error probability and provide results that are within a fraction of a dB from the actual values for error probability less than 10^{-1} for case 3 and 10^{-2} for case 4. The last bound (case 4) is especially useful since it does not involve Monte Carlo simulations to estimate the error probabilities for the cases involving two or three simultaneous CW jamming tones. From these figures we find that by employing Scheme II carrier jittering, the communicator gains approximately 3 dB compared to a system without carrier jittering and offers performance within one dB from the partial-band noise jamming case. This verifies the inspection made in [8] that a reasonable penalty on the jammer for not knowing the exact band structure of the communicator is about 3 dB.

VI. Conclusion

In this paper, expressions were derived for the probability of error for orthogonal BFSK signaling when the channel is corrupted by CW tone interference with arbitrary frequency offsets and AWGN. We observed that for certain values of frequency offsets, an interfering CW tone with very large power could have no effect on the performance of the system. Also for other values of offsets, interfering tones with power less than that of the intended signal could cause errors with significant probability. When the background noise is negligible, the probability of error was found to be highly dependent on the frequency offset and the interference power but is less so for the case when background noise is significant.

The performance of FHSS communications systems employing two different types of carrier jittering schemes under one-dimensional tone jamming were considered. We found that with an appropriate jittering scheme, the communicator gains about 3 dB compared to a system without carrier jittering and reduces the effectiveness of the one-dimensional tone jammer to within a fraction of the partial-band noise jammer.

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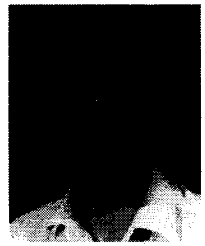

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