Trellis-Coded Direct-Sequence Spread-Spectrum Communications

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***Abstract*—This paper considers the application of trellis coding techniques to direct-sequence spread-spectrum multiple-access (DS/SSMA) communication. The unique feature of the trellis codes considered is that they are constructed over the set of possible signature sequences rather than over some standard 2-D signal constellation. The resulting codes have a small number of signals per dimension. We present several examples of these trellis codes, and suggest possible methods of implementation. We also present a detailed error analysis for this system, which employs techniques developed by Lehnert and Pursley to accurately model the multiple access interference. We generate numerical results for several examples and conclude that the proposed trellis coded systems yield significant performance improvements over binary antipodal DS/SSMA systems. In addition, the new trellis codes perform better than standard error control techniques with the same complexity and code rate. Analytic results are verified with simulations.**

I. Introduction

IN 1982, Ungerboeck [12] showed that considerable performance gains are possible in systems which combine the operations of coding and modulation. Since that time, the application of trellis codes to various communication systems has been the subject of considerable research. The effectiveness of coded-modulation techniques for communication over narrowband channels is well established. Trellis coding techniques have been successfully applied to telephone and satellite channels, where coding gains of 3–6 dB have been achieved with only modest complexity [12]. It is reasonable to believe that trellis coding will be useful on other channels as well.

In this paper, we examine the use of trellis coding techniques for DS/SSMA communication. Boudreau and Falconer have considered trellis codes constructed over an MPSK signal set for a DS/SSMA system [1], [2]. Boudreau and Falconer take a standard Ungerboeck type code for multiple phase shift keyed (MPSK) modulation and then use standard direct-sequence modulation; that is they then multiply the MPSK signal by a binary m-sequence to spread the signal over a large bandwidth. References [1], [2] report that this approach did not yield a performance advantage over standard convolutional codes combined with direct-sequence spread-spectrum.

Our approach is to construct the trellis code over the set of possible signature sequences. A standard direct-sequence system has a very large number of dimensions per signal. For example, the standard binary direct-sequence system has two antipodal signals spread into N dimensions using a sequence of length N (chips/bit). Our approach is to expand the number of possible spreading sequences used. A trellis is then used to allow only certain combinations that have large minimum distance. When we expand the number of sequences we decrease the minimum distance between sequences, but the trellis code increases the minimum distance of the code above that of the uncoded system. We expand the sequences from two antipodal sequences to a biorthogonal set of sequences. Enge and Sarwate in [3] considered using an expanded set of signature sequences but no additional coding. We note that for reasonably large sequence length, there is no shortage of available sequences. The difficulty is how to use the numerous sequences with good properties in a judicious manner. Although trellis codes are usually considered most useful when bandwidth is scarce, the results presented here show that they can produce substantial coding gains when used in a DS/SSMA system, with no expansion of bandwidth or reduction of data rate.

In certain special cases the codes we derive are identical to those of Boudreau and Falconer. However, in general our codes are different. We show in the numerical results section that for certain values of coding complexity and data rates, our codes are in fact superior to standard convolutional codes with direct-sequence. In comparing a trellis coded system with an uncoded system or a system that uses traditional codes in conjunction with spreading, it is important to make a fair comparison as far as possible. A folk theorem in the direct-sequence literature that should perhaps be called a myth is that traditional coding comes at no cost in terms of bandwidth expansion and the underlying processing gain of the spread spectrum system. To show why this is a myth, consider a direct-sequence spread-spectrum system for multiple-access using sequences of length N (chips/bit). The average cross-correlation magnitude between sequences for long sequences behaves (roughly) as $\frac{K}{\sqrt{N}}$ [8], [9]. The bandwidth expansion of the system is N. Suppose we add a convolutional code of rate 1/2. Then to maintain the same total bandwidth expansion the sequence length should be reduced by a factor of 2. This will increase significantly the average cross correlation magnitude.

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Of course, the convolutional code will be able to cope with some of the additional interference without causing an error. Nevertheless, it is clear that the addition of standard coding is not free, even though the bandwidth and data rate have not changed. In our approach the number of possible transmitted sequences is increased to allow redundancy to be added via coding. The sequences become closer but in conjunction with the trellis code the signals along two paths are actually farther apart than the baseline system. Furthermore, the sequence length need not be decreased. In our comparisons with either uncoded systems or coded systems we keep the bandwidth expansion, data rate, and coding complexity constant.

One simple way to view our approach to trellis coded direct-sequence spread-spectrum is to start with either an uncoded binary direct-sequence spread-spectrum system or a direct-sequence spread-spectrum system using orthogonal sequences. Expand the number of sequences to form a biorthogonal signal set. Then in designing the trellis when two paths diverge from some state, assign signals from the biorthogonal signal set that are antipodal and similarly when paths remerge.

In this paper, we undertake a rigorous performance analysis of these systems, using bit error probability as a performance measure. While it is fairly easy to obtain an estimate of the performance in white Gaussian noise with high signal-to-noise ratio (via the coding gain of a coded modulation system), it is more difficult to analyze the performance in a multiple-access environment. The analysis used is an extension of the technique introduced by Lehnert and Pursley in [4] and [5].

The remainder of this paper is organized as follows. In the following section we describe our trellis codes. In Section III we present a model for a DS/SSMA system with trellis coding. Two possible implementations of biorthogonal sets of signature sequences are illustrated. We describe the analytic techniques used for performance analysis in Section IV, and present numerical results in Section V. These results compare trellis coded system with both uncoded systems and with convolutionally coded systems of equivalent complexity. Analytic results are verified with simulations. Conclusions follow in Section VI.

II. Trellis Coded Spread-Spectrum

The operation of a trellis encoder may be conceptually divided into two stages. In the first stage, redundancy is added to input data by means of a convolutional code. In the second stage, the output of the convolutional encoder is mapped to a point in some constellation of possible output signals, and that signal point is transmitted. The idea is to choose the convolutional code and the mapping such that distinct paths through the trellis are separated by large squared Euclidean distance.

An often used figure of merit for trellis codes is the asymptotic coding gain G, given by

$$G = 10\log_{10}\left(\frac{d_{\text{free}}}{d_{\min}}\right)^2 \tag{1}$$

where $d_{\min}$ is the minimum Euclidean distance separating signal points of the baseline system and d_{free} is the minimum Euclidean distance separating distinct paths through the trellis. Throughout this paper, we take a binary antipodal DS/SSMA system as the baseline. Each of the codes considered here has a significant coding gain with respect to the appropriately chosen orthogonal system [3] as well as the binary antipodal system.

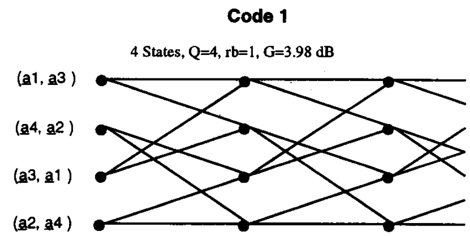

Fig. 1. 4 State trellis code over 4-ary biorthogonal signal set (code 1).

Most trellis codes are constructed over dense signal constellations which contain many signals per dimension. The MPSK and QASK signal sets are examples of such constellations. Boudreau and Falconer's codes are constructed over an MPSK signal set. It is also possible to construct a trellis code over relatively sparse signal sets. For example, consider a set of $Q/2$ orthogonal signal vectors $\{\mathbf{a}_j\}$, where

$$\mathbf{a}_i \perp \mathbf{a}_j, \quad i \neq j, i = 1, \cdots, \frac{Q}{2}. \tag{2}$$

Using this signal set, we could transmit $\log_2(Q) - 1$ bits per signaling interval. Now expand this signal set by a factor of two by letting

$$\mathbf{a}_{i+\frac{Q}{2}} = -\mathbf{a}_i, \quad i = 1, \cdots, \frac{Q}{2}. \tag{3}$$

This new biorthogonal signal set contains Q signal points in $Q/2$ dimensions. It could be used to transmit $\log_2(Q)$ bits per signaling interval. Alternatively, we could construct a trellis code which transmits information at rate $r_b = \log_2(Q) - 1$ bits per signaling interval. The trellis code operates in two stages. The first stage is a convolutional encoder of rate $\frac{\log_2(Q)-1}{\log_2(Q)}$. The second is a mapping from $\log_2(Q)$ bits to Q signal points.

Using Ungerboeck's idea of set partitioning, we have constructed trellis codes for biorthogonal signal sets with 4, 8, and 16 signal points. Two possible techniques for implementing a biorthogonal set of signal constellation are presented in Section III. Trellis codes can be constructed for DS/SSMA by partitioning the set of signature sequences in the same manner as signal constellations are partitioned in [12]. A simple 4 state trellis code which uses this signal set partitioning is also shown in Fig. 1. We will refer to this as Code 1. The 4-ary biorthogonal signal constellation is geometrically the same as a 4-PSK constellation, although it is implemented through the choice of signature sequences. We will also present numerical results for codes contructed over 8-ary and 16-ary biorthogonal signal sets. We will refer to these examples as Code 2, and Code 3 respectively. In general parallel transitions between states are possible.

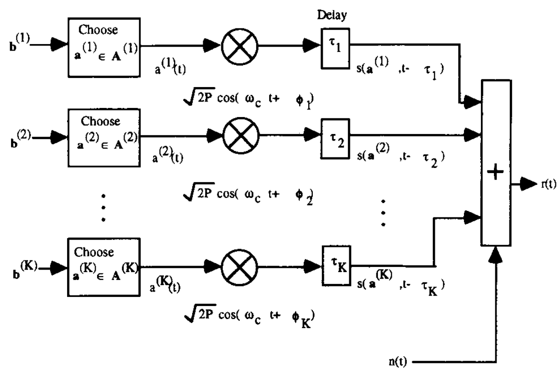

Fig. 2. DS/SSMA system model.

III. System Model

In this section, we describe a model for a DS/SSMA system, based on [6]. We have generalized this standard model beyond systems which employ uncoded binary antipodal signaling. We describe the transmitter, channel, and receiver for a generalized DS/SSMA system.

During each signaling interval, transmitter k receives a vector $\mathbf{b}^{(k)}$ of r_b bits. The transmitter maps $\mathbf{b}^{(k)}$ into one of Q signature sequences $\mathbf{a}^{(k)} \in \mathbf{A}^{(k)}$, where $\mathbf{a}^{(k)} = \{\mathbf{a}_1^{(k)}, \cdots, \mathbf{a}_Q^{(k)}\}$. This mapping may be fixed over time, as is in a conventional binary antipodal system, or the mapping may depend on past input data, as it does in our proposed trellis coding systems. There are numerous classes of pseudorandom sequences with desirable correlation properties from which to assign $\mathbf{a}^{(k)}$ [7].

A DS/SSMA transmitter with K simultaneous users is depicted in Fig. 2. During each signaling interval of duration T, user k's transmitter selects a signature sequence $\mathbf{a}_i^{(k)}$ from a set $\mathbf{A}^{(k)}$ of possible signature sequences. The signature sequence consists of N chips of duration T_c, which take values on the set $\{+1, -1\}$. The signature sequence is modulated by a carrier waveform with frequency ω_c, power P, and phase ϕ_k. User k transmits the signal $s(\mathbf{a}_i^{(k)}, t)$, where

$$s\left(\mathbf{a}_i^{(k)}, t\right) = \sqrt{2P}\cos(\omega_c t + \phi_k) \sum_{n=0}^{N-1} a_{i,n}^{(k)} p_{T_c}(t - nT_c). \quad (4)$$

The unit pulse function $p_{T_c}(\cdot)$ takes the value 1 on the interval $[0, T_c)$, and the value 0 elsewhere. As a result, the signal tranmsmitted by user k for $t \in (-\infty, \infty)$ is $s_k(t) = \sum_{j=-\infty}^{\infty} s(\mathbf{a}_{p(j)}^{(k)}, t - jT)$, where $p(j)$ is the index of the signature sequence transmitted during the interval $[(j-1)T, jT)$.

The received signal $r(t)$ is the sum of K delayed signals and Gaussian noise signal $n(t)$ with two-sided power spectral density $N_0/2$ [6]

$$r(t) = n(t) + \sum_{k=1}^{K} s_k(t - \tau_k), \quad (5)$$

where each signal is delayed by τ_k, uniformly distributed on $[0, T)$. It is convenient to write $\tau_k = \gamma_k T_c + \Delta_k$, where $\gamma_k = \lfloor \frac{\tau_k}{T_c} \rfloor$ and the floor function $\lfloor x \rfloor$ is the greatest integer no larger than x. It follows that $\Delta_k = \tau_k - \gamma_k T_c$, where γ_k is uniformly distributed on the set $\{0, \cdots, N-1\}$, and Δ_k is uniformly distributed on the interval $[0, T_c)$. The received signal from user k now has phase $\theta_k = \phi_k - \omega_c \tau_k (\text{mod } 2\pi)$, with θ_k is uniformly distributed on $[0, 2\pi)$.

A correlation receiver is assumed throughout this paper, although our trellis codes may be used in conjunction with any DS/SS receiver. The received signal $r(t)$ is demodulated by a signal which is synchronized in phase and delay to the signal from user k. We assume synchronization with user 1 and therefore $\theta_1 = \tau_1 = 0$ with no loss of generality. At the output of the receiver, a set of Q decision statistics $\{Z_1, \cdots, Q_Q\}$ is generated, where

$$Z_i = \int_0^T r(t)\cos(\omega_c t) \sum_{n=0}^{N-1} a_{i,n}^{(k)} p_{T_c}(t - nT_c)dt, \quad i = 1, \cdots, Q. \quad (6)$$

In a trellis coded system, the receiver uses the set of decision statistics as metrics in the Viterbi decoder which selects the path associated with the largest cumulative decision statistic.

In order to implement one of the trellis codes described, it is necessary to construct a biorthogonal set of signature sequences for each user. We describe two such constructions here. Let $\mathbf{a}$ be any sequence of length N where N is a power of 2, and let H be an $N \times N$ Haddamard matrix. Each row of the Haddamard matrix is orthogonal to every other row of the matrix. Let $h_{i,j}$ be the ijth entry of the Haddamard matrix, and let the vector $\mathbf{a}_i$ be defined as

$$\mathbf{a}_i = (a_{1,0}h_{i,0}, \cdots, a_{1,N-1}h_{i,N-1}), \quad i = 0, \cdots, N-1. \quad (7)$$

Then $\{\mathbf{a}_i\}$ is a set of N orthogonal sequences. We select a set of $Q/2$ orthogonal sequences from that set, and add the $Q/2$ antipodes of those sequences to construct a Q-ary biorthogonal set.

It is also possible to construct an approximately biorthogonal signal set, starting from the AO/LSE m-sequences which have desirable auto-correlation and cross-correlation properties [7]. For $N = 31$ and $N = 63$ there are six such sequences. Define the left vector rotation operator T by

$$T = (\mathbf{a} = (a_0, a_1, \cdots, a_{N-1})) = (a_1, \cdots, a_{N-1}, a_0). \quad (8)$$

Then the m-sequence $\mathbf{a}$ exhibits the property that all rotations of $\mathbf{a}$ are approximately orthogonal to $\mathbf{a}$ in the sense that

$$\sum_{n=0}^{N-1} a_n T^i(\mathbf{a})_n = -1, \quad i = 1, \cdots, N-1. \quad (9)$$

Therefore, we can construct approximately biorthogonal signal sets for K different transmitters as follows. We let $\mathbf{a}_i^{(k)}$ equal one of the AO/LSE m-sequences for $k = 1, \cdots, K$. The next $Q/2 - 1$ signature sequences in $\mathbf{A}^{(k)}$ are rotations of $\mathbf{a}_i^{(k)}$

$$\mathbf{a}_i^{(k)} = T^{i-1}(\mathbf{a}_1^{(k)}), \quad i = 2, \cdots, \frac{Q}{2}, \quad k = 1, \cdots, K. \quad (10)$$

The final $Q/2$ signature sequences in each user's set are antipodal to the first $Q/2$ sequences. Note that the bandwidth of the resulting trellis coded DS/SSMA signal is no greater than the bandwidth of a binary antipodal DS/SSMA signal, but the cross-correlation functions between any two members of different signal sets have exactly the same distribution as the cross-correlation functions of the AO/LSE m-sequences upon which the signal set is based.

IV. Performance Analysis

In this paper, we conduct a careful performance analysis of the trellis coded DS/SSMA system first introduced in [10]. We consider the probability of bit error P_b which is closely related to the first event error probability P_e. Let p be the correct path through the trellis and let $\tilde{p}$ be any path through the trellis which diverges from p during the first time interval and remerges with p after a finite number of time intervals. Let Z_p and $Z_{\tilde{p}}$ be the cumulative decision statistics associated with paths p and $\tilde{p}$ by the Viterbi decoder. Finally, let $\{d_1, d_2, \cdots,\}$ be the set of all distances separating distinct paths through the trellis, where $d_1 < d_2 < d_3 < \cdots$. Clearly, $d_1 = d_{\text{free}}$. A union bound [14] allows us to write expressions for P_e and P_b

$$P_e \leq \sum_{i=1}^{\infty} \sum_{\{\tilde{p}:\, ||p-\tilde{p}||=d_i\}} \Pr(Z_{\tilde{p}} > Z_p), \tag{11}$$

$$P_b \leq \sum_{i=1}^{\infty} \sum_{\{\tilde{p}:\, ||p-\tilde{p}||-d_i\}} W_{\tilde{p}} \Pr(Z_{\tilde{p}} > Z_p) \tag{12}$$

where $W_{\tilde{p}}$ is the average number of bit errors caused by choosing path $\tilde{p}$. For moderate SNR, error events at short distance dominate the performance of the trellis code. For this reason, it is reasonable to truncate the summation in (12) to include only a few values of d_i or even a single value. We concentrate our efforts on deriving tight exact bounds on $\Pr(Z_{\tilde{p}} > Z_p)$, the probability that the Viterbi decoder will select path $\tilde{p}$ over the correct path p. This enables us to compute as many terms of the union bound as necessary to obtain accurate performance results. We attack this problem with a technique developed by Lehnert and Pursley. This technique is introduced in [4] and discussed in more detail for the case of deterministic sequences in [5]. Although our basic approach is identical to the approach of [4] and [5], the details will differ because we consider coded systems with signature sequences which are not necessarily antipodal.

A. The Error Event $\tilde{p}$

To assist our analysis, we employ the conceptual device of a correlation receiver designed to distinguish solely between the divergent paths p and $\tilde{p}$ through the trellis. Suppose that paths p and $\tilde{p}$ diverge for L_p consecutive time intervals before remerging. Define the index $p(j)$ such that a transmitter following path p uses the signature sequence with index $p(j)$ during the time interval j. Define $\tilde{p}(j)$ analogously. Path p consists of the successive transmission of signature sequences: $\mathbf{a}_{p,(1)}^{(1)}, \mathbf{a}_{p,(2)}^{(1)}, \cdots, \mathbf{a}_{p(L_p)}^{(1)}$. We define the extended signature sequences $\mathbf{s}_p$ and $\mathbf{s}_{\tilde{p}}$ as the concatenation of the transmitted signature sequences

$$\begin{aligned}\mathbf{s}_p &= (s_{p,0}, \cdots, s_{p,L_pN-1}) \\ &= \left(a_{p(1),0}^{(1)}, \cdots, a_{p(1),N-1}^{(1)}, \cdots, a_{p(L_p),0}^{(1)}, \cdots, a_{p(L_p),N-1}^{(1)}\right),\end{aligned} \tag{13}$$

$$\begin{aligned}\mathbf{s}_p &= (s_{\tilde{p},0}, \cdots, s_{\tilde{p},L_pN-1}) \\ &= \left(a_{\tilde{p}(1),0}^{(1)}, \cdots, a_{\tilde{p}(1),N-1}^{(1)}, \cdots, a_{\tilde{p}(L_p),0}^{(1)}, \cdots, a_{\tilde{p}(L_p),N-1}^{(1)}\right).\end{aligned} \tag{14}$$

Now define the difference signature sequence $\mathbf{x}$ by

$$x_n = \frac{1}{2}(s_{p,n} - s_{\tilde{p},n}), \quad n = 0, \cdots, L_pN - 1. \tag{15}$$

For each n, x_n takes one of the three values in $\{+1, -1, 0\}$. Let the effective signature sequence length $N_e = \sum_{n=0}^{L_pN-1} |x_n|$ be the number of chips in which $\mathbf{s}_p$ differs from $\mathbf{s}_{\tilde{p}}$. N_e is related to the Euclidean distance separating paths p and $\tilde{p}$ by $||p - \tilde{p}||^2 = PT_cN_e$.

To further simplify notation, we also define extended signature sequences for each of the multiple-access interferers. Because they are delayed by $\{\tau_k\}$, the multiple access interference will be composed of signature sequences from $L_p + 1$ consecutive intervals. The extended signature sequence $\mathbf{s}^{(k)}$ will be the concatenation of these $L_p + 1$ signature sequences

$$\mathbf{s}^{(\mathbf{k})} = (s_{-N}^{(k)}, \cdots, s_{L_pN-1}^{(k)}), \quad k = 2, \cdots, K. \tag{16}$$

During the time interval $[0, L_pT)$, user 1 will transmit the signal $s(\mathbf{s}_p, t)$, where

$$s(\mathbf{s}_p, t) = \sqrt{2P} \cos(\omega_c t) \sum_{n=0}^{L_pN-1} s_{p,n} p_{T_c}(t - nT_c) \tag{17}$$

and during the time interval $[-T, L_pT)$, users 2 through K will transmit the signal $s(\mathbf{s}^{(k)}, t)$, where

$$\begin{aligned}s(\mathbf{s}^{(k)}, t) &= \sqrt{2P} \cos(\omega_c t + \phi_k) \sum_{n=-N}^{L_pN-1} s_n^{(k)} p_{T_c}(t - nT_c), \\ k &= 2, \cdots, K.\end{aligned} \tag{18}$$

The received signal $r(t)$ during the time interval $[0, L_pt)$, may be written

$$r(t) = n(t) + s(\mathbf{s}_p, t) + \sum_{k=2}^{K} s_k\left(\mathbf{s}^{(k)}, t - \tau_k\right). \tag{19}$$

Now consider a decision statistic $Z_{p\to\tilde{p}}$ which has the sole function of distinguishing between the paths p and $\tilde{p}$. The decision statistic $Z_{p\to\tilde{p}}$ is determined from the correlation

$$Z_{p\to\tilde{p}} = \int_0^{L_pT} r(t) \cos(\omega_c t) \sum_{n=0}^{L_pN-1} x_n p_{T_c}(t - nT_c)\, dt. \tag{20}$$

We can decompose $Z_{p\to\tilde{p}}$ into three components by writing

$$Z_{p\to\tilde{p}} = \xi + I_{1,1}(\mathbf{s}_p, 0, 0) + \sum_{k=2}^{K} I_{k,1}\left(\mathbf{s}^{(k)}, \theta_k, \tau_k\right) \tag{21}$$

where

$$\xi = \int_0^{L_pT} n(t)\cos(\omega_c t)\sum_{n=0}^{L_pN-1} x_n p_{T_c}(t-nT_c)dt \quad (22)$$

and

$$I_{k,1}\left(\mathbf{s}^{(k)},\theta_k,\tau_k\right) = \int_0^{L_pT} s\left(\mathbf{s}^{(k)},t-\tau_k\right)\cos(\omega_c t) \times \sum_{n=0}^{L_pN-1} x_n p_{T_c}(t-nT_c)dt. \quad (23)$$

In (21) ξ represents the contribution of the noise $n(t)$ at the receiver, $I_{1,1}$ represents the desired signal component, and $I_{k,1}$ represents the contribution of the multiple access interference from user k at user 1's receiver for $k > 1$. We will freely shorten $I_{k,1}(\mathbf{s}^{(k)},\theta_k,\tau_k)$ to $I_{k,1}(\mathbf{s},\theta,\tau)$ or just $I_{k,1}$ whenever the parameters are clear from the context.

Since $n(t)$ is a Gaussian random process, ξ is also a Gaussian random variable. Making the usual assumption that the double frequency terms in (22) can be ignored, we calculate that ξ mean 0 and variance $\frac{N_0N_eT_c}{4}$. Similarly, we find that the contribution of the desired signal is

$$I_{1,1} = \sqrt{\frac{P}{2}}N_eT_c. \quad (24)$$

The expression $\sqrt{\frac{P}{2}}T_c$ will appear frequently so henceforth we will write $A_c = \sqrt{\frac{P}{2}}T_c$. Thirdly, we derive an expression for $I_{k,1}$, the multiple access interference from user k at user 1's receiver. Substituting (18) into (23) and simplifying yields

$$I_{k,1}\left(\mathbf{s}^{(k)},\theta_k,\tau_k\right) = \sqrt{\frac{P}{2}}\cos(\theta_k)\left[\Delta_k\sum_{n=0}^{L_pN-1} s_{n-\gamma_k-1}^{(k)}x_n + (T_c-\Delta_k)\sum_{n=0}^{L_pN-1} s_{n-\gamma_k}^{(k)}x_n\right]. \quad (25)$$

We have expressed the decision statistic $Z_{p\to\tilde{p}}$ as the sum of three types of terms: Gaussian noise, desired signal, and multiple access interference. If the multiple access interference were a constant, the problem would reduce to one of binary detection in Gaussian noise. This is not the case; however, we will bound the probability that $I_{k,1}$ lies within an arbitrarily small interval.

B. The Normalized Synchronous Interference $J_{k,1}$

We focus on the statistical characterization of the multiple access interference $I_{k,1}$. It is convenient to write

$$I_{k,1}\left(\mathbf{s}^{(k)},\theta_k,\tau_k\right) = A_c\cos(\theta_k)J_{k,1}\left(\mathbf{s}^{(k)},\tau_k\right) \quad (26)$$

where

$$J_{k,1}(\mathbf{s}^{(k)},\tau) = \frac{\Delta}{T_c}\sum_{n=0}^{L_pN-1} s_{n-\gamma-1}^{(k)}x_n + \frac{T_c-\Delta}{T_c}\sum_{n=0}^{L_pN-1} s_{n-\gamma}^{(k)}x_n. \quad (27)$$

$J_{k,1}$ does not depend on the random phase θ so we may think of it as a normalized, synchronous version of the multiple access interference. Shortly we will show that the probability distribution of the random variable $J_{k,1}$ assumes a remarkably simple form.

Now define the set $B = \{n: 0 \le n \le L_p\ N-1, x_n \ne 0\}$, where $|B| = N_e$. We rewrite (27) as

$$J_{k,1}(\mathbf{s}^{(k)},\tau) = \frac{\Delta}{T_c}\sum_{n\in B} s_{n-\gamma-1}^{(k)}x_n + \frac{T_c-\Delta}{T_c}\sum_{n\in B} s_{n-\gamma}^{(k)}x_n. \quad (28)$$

Now define the nonintersecting sets $B_1 = \{n \in B: s_{n-\gamma-1} = s_{n-\gamma}\}$ and $B_{-1} = \{n \in B: s_{n-\gamma-1} = -s_{n-\gamma}\}$. Noting that $B_1 \cup B_{-1} = B$, we rewrite (8) as

$$J_{k,1}(\mathbf{s},\tau) = \frac{\Delta}{T_c}\sum_{n\in B_1} S_{n-\gamma}x_n + \frac{T_c-2\Delta}{T_c}\sum_{n\in B_{-1}} s_{n-\gamma}^{(k)}x_n. \quad (29)$$

This is the form that we want. The next proposition demonstrates how we may condition $J_{k,1}$ on a certain index variable so that it assumes a simple "staircase" of values. This proposition is a straightforward extension of results from [4].

Proposition 1: Let $J_{k,1}$ be defined as in (27). Then $J_{k,1}$ can be conditioned on a discrete random variable Λ_k which takes values on the set $\{-N_e, 1-N_e, \cdots, N_e\}$, such that

$$\Pr[J_{k,1} = \lambda|\Lambda_k = \lambda] = 1, \quad \lambda = -N_e, 2-N_e, \cdots, N_e, \quad (30)$$

$$p_{J_{k,1}|\Lambda_k}(x \mid \Lambda_k = \lambda) = \frac{1}{2}p_2(x-\lambda+1), \quad \lambda = 1-N_e, 3-N_e, \cdots, N_e-1$$

where the function $p_2(\cdot)$ is the unit pulse function of duration 2. That is, when $N_e - \lambda$ is even, the random variable $J_{k,1}$ equals λ with probability one, and when $N_e - \lambda$ is odd, the random variable $J_{k,1}$ is uniformly distributed on the range $[\lambda-1, \lambda+1]$.

The proof of Proposition 1 is given in the Appendix. This proof, which defines discrete random variables P_k and Q_k, indicates one method for computing the probability distribution of the index variable Λ_k. First, we compute the joint probability mass function of P_k and Q_k, averaging over all possible values for the interfering sequence $\mathbf{s}^{(k)}$ and all possible delays γ_k

$$p_{P_kQ_k}(i,j) = \sum_{s^{(k)}\in[\mathbf{A}^{(k)}]^{L_p+1}}\sum_{\gamma_k=0}^{N-1}\frac{1}{N} \times \Pr(\mathbf{s}^{(k)})I\left(P_k = i, Q_k = j \mid \mathbf{s}^{(k)},\gamma_k\right) \quad (31)$$

where the indicator function $I(\cdot)$ is given by

$$I(P_k = i, Q_k = j \mid \mathbf{s}^{(k)},\gamma_k) = \begin{cases}1, & \sum_{n\in B_i} s_{n-\gamma}x_n = i,\ \sum_{n\in B_{-1}} s_{n-\gamma}x_n = j\\ 0, & \text{otherwise}\end{cases} \quad (32)$$

Our trellis codes exhibit the property that all valid paths through the trellis are equally likely which simplifies the calculation of $\Pr(\mathbf{s}^{(k)})$ in (31). For even values of $N_e - \Lambda$, we find that

$$p_{\Lambda_k}(\lambda) = p_{P_kQ_k}(\lambda, 0), \quad \lambda = -N_e, 2-N_e, \cdots, N_e. \quad (33)$$

The pmf of Λ is more difficult to compute for $N_e - \Lambda$ odd, but a straightforward calculation yields

$$p_{\Lambda_k}(\lambda) = \sum_{i=-N_e}^{N_e} \times \left[\sum_{j=|i-\lambda|+1}^{N_e} \frac{1}{j} p_{P_kQ_k}(i,j) + \sum_{j=-N_e}^{-|i-\lambda|-1} \frac{1}{|j|} p_{P_kQ_k}(i,j)\right]$$
$$\lambda = 1-N_e, 3-N_e, \cdots, N_e - 1. \quad (34)$$

The calculations can be simplified by exploiting symmetries in the distribution of P_k, Q_k, and Λ_k.

C. Modeling the Distribution of $I_{k,1}$

Having described the conditional probability distribution of $J_{k,1}$, we now describe the conditional probability distribution of $I_{k,1}$. In particular, we want to compute the probability that $I_{k,1}$ lies within any arbitrarily small closed interval. The next proposition accomplishes this.

Proposition 2: Let $I_{k,1}$ be related to $J_{k,1}$ by (22). Let the random variable Λ_k be the same random variable described in Proposition 1, and let $[a, b]$ be any closed interval on the real line. Then (35), the top equation at the bottom of this page, where

$$F(x;\lambda) = \begin{cases} 0, & x \le -\lambda \\ \frac{1}{2} + \frac{1}{\pi}\arcsin\left(\frac{x}{\lambda}\right), & |x| < \lambda \\ 1, & x \ge \lambda, \end{cases} \quad (36)$$

and (37), the bottom equation at the bottom of this page. The proof of Proposition 2 follows from [5]. Equations (36) and (37) are equivalent to (19) and (20), respectively, in [5].

Following the example of [4] and [5], we use the results of Proposition 2 to approximate the probability distribution of $I_{k,1}$. For each λ in the set $\{-N_e, \cdots, N_e\}$, we define a $2N_u + 1$ component vector $\mathbf{d}(\lambda)$ which models the probability distribution of the random variable $I_{k,1}$, conditioned on $\Lambda_k = \lambda$. Each component of the vector is defined by

$$d_j(\lambda) = \Pr\left\{I_{k,1} \in \left[\frac{j-\frac{1}{2}}{N_u}A_cN_e, \frac{j+\frac{1}{2}}{N_u}A_cN_e\right] \mid \Lambda_k = \lambda\right\},$$
$$j = -N_u, \cdots, N_u. \quad (38)$$

The components of $\mathbf{d}(\lambda)$ can be computed using (35). Now for each interferer k, we define a $2N_u + 1$ component vector $\mathbf{d}^{(k)}$ which models the unconditional probability distribution of the interference $I_{k,1}$. We compute each component of $\mathbf{d}^{(k)}$, by taking the expectation of $\mathbf{d}(\lambda)$ over the distribution of Λ_k

$$\mathbf{d}^{(k)} = \sum_{\lambda=-N_e}^{N_e} p_{\Lambda_k}(\lambda)\mathbf{d}(\lambda), \quad k = 2, \cdots, K. \quad (39)$$

We have now modeled the continuous probability density function of $I_{k,1}$ as a vector with $2N_u + 1$ components. N_u is the number of intervals into which we have divided the positive portion of the density function. Large N_u implies greater accuracy, as well as larger computational effort.

D. Bounds on Error Event Probability

We can now compute tight bounds on the probability of an error event of any distance. Recall from (21) that we need to compute the probability distribution of the total multiple access

$$\Pr[I_{k,1} \in [a,b] \mid \Lambda = \lambda] = \begin{cases} F\left(\frac{b}{A_c};\lambda\right) - F\left(\frac{a}{A_c};\lambda\right), & \lambda = -N_e, 2-N_e, \cdots, N_e \\ G\left(\frac{b}{A_c};\lambda\right) - G\left(\frac{a}{A_c};\lambda\right), & \lambda = 1-N_e, 3-N_e, \cdots, N_e - 1, \end{cases} \quad (35)$$

$$G(x;\lambda) = \begin{cases} 0, & y \le -\lambda - 1 \\ \frac{1}{2} + \frac{1}{2\pi}\Big\{y \ln\left[\frac{\lambda+1+\sqrt{(\lambda+1)^2-y^2}}{\lambda-1+\sqrt{(\lambda-1)^2-y^2}}\right] \\ \quad +(\lambda+1)\arcsin\left(\frac{y}{\lambda+1}\right) \\ \quad -(\lambda-1)\arcsin\left(\frac{y}{\lambda-1}\right)\Big\}, & 0 \le |y| \le \lambda - 1 \\ \frac{1}{2} + \frac{1}{2\pi}\Big\{y \ln\Big[\lambda+1 \\ \quad +\sqrt{(\lambda+1)^2-y^2}\Big] + (\lambda+1)\arcsin\left(\frac{y}{\lambda+1}\right) \\ \quad +\frac{(\lambda-1)\pi y}{2|y|} - y\ln|y|\Big\}, & \lambda - 1 \le |y| \lambda + 1 \\ 1, & y > \lambda + 1. \end{cases} \quad (37)$$

interference $\sum_{k=2}^{K} I_{k,1}$. Note that each random variable $I_{k,1}$ is independent of all other $I_{k',1}$, conditioned on the difference sequence $\mathbf{x}$. Since the difference sequence $\mathbf{x}$ is fixed, each $I_{k,1}$ is independent. The probability density of the sum of independent random variables is just the convolution of the individual probability density functions.

We define a $2(K-1)N_u+1$ component vector $\mathbf{D}$ which represents the probability distribution of the total multiple access interference $\sum_{k=2}^{K} I_{k,1}$. We compute $\mathbf{D}$ by $K-2$ discrete convolutions as follows

$$\mathbf{D} = \mathbf{d}^{(2)} \otimes \mathbf{d}^{(3)} \otimes \cdots \otimes \mathbf{d}^{(K)}. \tag{40}$$

We have computed the probability that the multiple access interference lies within a small interval. If the multiple access interference assumes some fixed value within that interval, then we have a problem of binary detection in Gaussian noise. The next proposition describes how we may compute the probability of an error event.

Proposition 3: Let $Z_{p\to\tilde{p}}$ be the decision statistic from a correlation receiver designed to distinguish between paths p and $\tilde{p}$ through the trellis. Then the probability of an error event may be upper and lower bounded by

$$\Pr[Z_{p\to\tilde{p}} < 0] \le \sum_{j=-N_u}^{N_u} D_j \mathbf{Q}\left\{\sqrt{\frac{2PT_cN_e}{N_0}}\left(1+\frac{2j+1-K}{2N_u}\right)\right\} \tag{41}$$

$$\Pr[Z_{p\to\tilde{p}} < 0] \ge \sum_{j=-N_u}^{N_u} D_j \mathbf{Q}\left\{\sqrt{\frac{2PT_cN_e}{N_0}}\left(1+\frac{2j+K-1}{2N_u}\right)\right\}. \tag{42}$$

The proof of Proposition 3 is given in [14]. The bounds on the probability of any specific error event which are given in (41) and (42) may be used in (11) and (12) to compute the error event probability and bit error probability, respectively.

V. Numerical Results

In this section, we apply the techniques developed in Section IV to generate numerical results for the performance of trellis coded DS/SSMA systems. From the preceding discussion, it is evident that the effective signature sequence length N_e is an important parameter. Our trellis codes produce performance gains over the corresponding binary antipodal DS/SSMA system due to two factors. First, the effective signature sequence length N_e is usually greater than the number of chips per time interval N. Second, because the trellis coded system transmits r_b bits per interval instead of just one, it allows a basic signature sequence of r_bN chips per interval to be selected with no increase in chip rate or required bandwidth. In this section, we present numerical results comparing the performance of trellis coded DS/SSMA systems with their binary antipodal counterparts, using the techniques described in Section IV.

A. Error Probabilities

Figs. 3 and 4 display plots of bit error probability versus SNR for uncoded binary antipodal DS/SSMA, and for a system employing Code 1. These plots were generated using the technique for evaluating error event probability described in Section IV, and truncating the summation in (11) to include only those paths at distance d_{free} from the correct path. The number of users ranges from 2 to 6. The signature sequence lengths are adjusted to keep N/r_b as close to constant as possible. The signature sequences for the binary antipodal system and the system using Code 1 are based on the set of AO/LSE m-sequences of length $N=31$. Comparison of Figs. 3 and 4 leads to the conclusion that the coded outperforms the uncoded system.

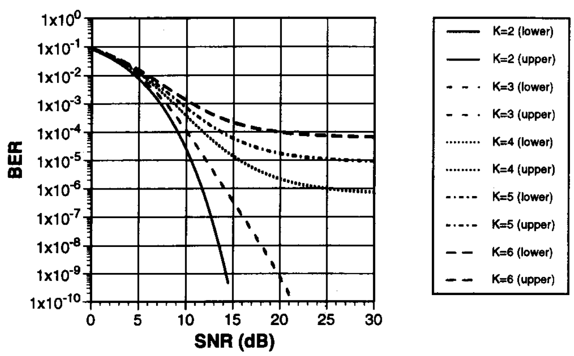

Fig. 3. Upper and lower bounds on bit error rate as a function of SNR for uncoded system.

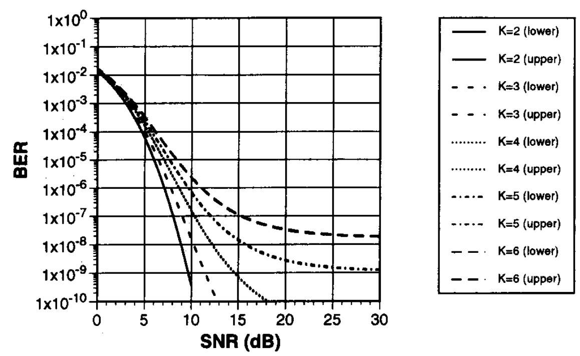

Fig. 4. Upper and lower bounds on bit error rate as a function of SNR for a system employing code 1.

A useful technique for comparing systems with different bit and chip rates is to define the normalized offered traffic G, where

$$G = \frac{r_bK}{N}\left(\frac{\text{User-bits}}{\text{unit bandwidth}}\right). \tag{43}$$

Comparing performance between systems with identical G guarantees a fair comparison between systems. Two additional codes are provided as examples: a 4 state code with $r_b=2$ and $G=3.01$ dB constructed over an 8-ary biorthogonal signal set which we label Code 2, and an 8 state code with $r_b=3$ and $G=4.77$ dB constructed over a 16-ary biorthogonal signal set which we designate Code 3. Fig. 5 displays a plot of the bit error probability versus G for fixed SNR for three Codes 1, 2 and 3.

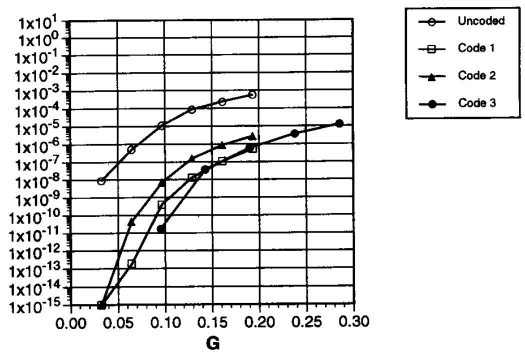

Fig. 5. Bit error rate as a function of normalized offered traffic for codes 1, 2, and 3.

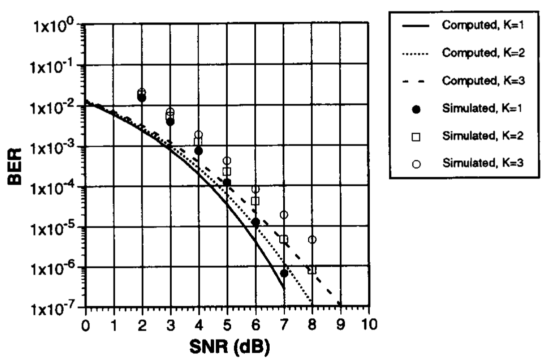

Fig. 6. Comparison of simulations with analytic results for system employing code 1.

B. Simulation Results

In order to evaluate the accuracy of our numerical calculations, we have performed simulations of the coded systems using the model of Section II, with pseudorandom variables for data, noise, phases and delays. We investigate the accuracy of our assumption that the minimum distance error event dominates the performance of the coded systems. Results of our simulation for Code 1 are plotted in Fig. 6 for SNR ranging from 2 to 8 dB, and for 1 through 3 multiple access users. Only those points which are within 20 percent of the actual value with 90 percent confidence are shown. For low SNR, the performance of the coded system is not as good as predicted in Section IV-D. However, even at these SNR, the coded system substantially outperforms the uncoded system. Moreover, the performance is seen to converge to the values predicted in Section IV-D as the SNR increases. At an SNR of 8 dB, the performance is within 1 dB of the predicted value. Simulations of Codes 2 and 3 demonstrate similar results.

C. Comparison With Convolutional Codes

The results of Subsections A and B above demonstrate that the trellis codes considered in this paper compare favorably with uncoded DS/SSMA. Another important basis of comparison is with other practical coding systems of similar complexity. In [2], Boudreau and Falconer discovered that the performance of standard Ungerboeck trellis codes

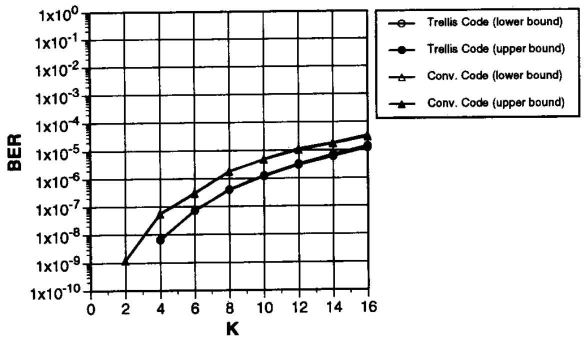

Fig. 7. Comparison of bit error rate versus K for code 1 and a 4 state convolutional code.

constructed over a PSK signal constellation was disappointing in comparison to soft decision convolutional codes in a DS/SS environment. As a result, we compare the performance of the trellis codes considered here with soft-decision convolutional codes of equivalent complexity.

Fig. 7 compares the performance of Code 1 with a 4 state convolutional code of rate $1/2$. Both codes have four states. However, since the trellis coded system has rate $r_b = 1$, it can employ a signature sequence of length twice that of a system with are rate $1/2$ convolutional code. In order to hold N/r_b constant, the trellis coded system has $N = 64$ while the system with convolutional coding has $N = 32$. The SNR is fixed at 12 dB. The results show that for any fixed bit error rate, the system with trellis coding is able to accommodate a greater number of multiple access users K. Similar results can be shown for Codes 2 and 3.

D. Discussion

The performance improvement is a result of two factors: the distance properties of the code and cross-correlation properties of the signature sequences. The larger signal set allows us to choose a longer signature sequence which can have better correlation properties. A comparison of various coded signaling schemes for DS/SSMA is given in Table I. The mean square value of the multiple access interference term $I_{2,1}$ is of interest. Note that the trellis coded systems have lower values of $E[I_{2,1}^2]$ than all other coding schemes with similar distance properties.

For the sake of brevity, we have presented results only for the deterministic implementation described in Section II-A. We have carried out a similar performance analysis for the case of randomly generated signature sequences. Those results also indicate that significant performance improvements are obtainable using a coded modulation approach.

VI. Conclusion

In this paper we have described a proposed implementation for trellis coded DS/SSMA communication. We have carried out an extensive error analysis based on the techniques introduced in [4], [5]. The proposed trellis coded systems outperform the binary antipodal systems with equivalent bandwidth. This performance improvement stems from two factors. First, the trellis codes increase the Euclidean distance separating dis-

TABLE I
COMPARISON OF SELECTED SOFT DECISION CODES

Code	Signal Set	States	N	d^2_{free} (1)	N free	r_b (2)	$E[I_{k,1}]^2$
Uncoded	Antipodal	1	31	31	1	1	11.79
Uncoded	2-ary Orthogonal	1	31	16	1	1	7.98
Uncoded	4-ary Orthogonal	1	63	32	3	2	14.35
Uncoded	4-ary Biorthogonal	1	63	31	2	2	15.97
Trellis	4-ary Biorthogonal	2	31	47	1	1	21.80
Trellis (Code 1)	4-ary Biorthogonal	4	31	79	1	1	36.08
Trellis (Code 2)	8-ary Biorthogonal	4	63	63	1	2	20.67
Trellis	8-ary Biorthogonal	8	63	94	5	2	37.89
Convolutional	Antipodal	2	15	47	1	1/2	25.55
Convolutional	Antipodal	2	15	79	1	1/2	44.09
(8,4) Ext. Hamming	Antipodal	4(3)	15	63	14	1/2	71.21

(1) We have set $\sqrt{\frac{P}{2}}T_c = 1$.

(2) Data Rate = 1 bit per 31 chips.

(3) The (8,4 Extended Hamming Code can be decoded with a 4 state Viterbi Decoder.

tinct transmissions. Second, the higher bit rate, which allows for longer signature sequences with no additional expansion of bandwidth, leads to improved cross-correlation properties.

Although convolutional codes can be found with distance properties similar to these trellis codes, by viewing the codes as trellis codes we are able to use signature sequences with a longer period, and thus have superior cross-correlation properties. These superior cross-correlation properties can lead to a larger multiple access capability for systems employing trellis coding.

We are now examining the use of coded modulation with other receivers. In an effort to combat the well known near/far problem, several alternatives to the correlation receiver assumed in this paper have been proposed [13]. We are examining the use of these receivers for our coded modulation schemes.

APPENDIX

Proof of Proposition 1: Rewrite (29) in the form

$$J_{k,1} = P_k + \frac{T_c - 2\Delta}{T_c} Q_k \tag{44}$$

where

$$P_k = \sum_{n \in B_1} S_{n-\gamma} x_n, \tag{45}$$

$$Q_k = \sum_{n \in B_{-1}} S_{n-\gamma} x_n. \tag{46}$$

Since $P_k + Q_k$ is the sum of a total of N_e nonzero binary digits, it follows that $|P_k| + |Q_k| \leq N_e$, and therefore that $|J_{k,1}| \leq N_e$.

First assume that Q_k is even. This implies that P_k is even whenever N_e is even, and that P_k is odd whenever N_e is odd. In any case, $N_e - P_k$ will be an even integer. If $Q_k = 0$, then $J_{k,1} = P_k = \lambda$, where $N_e - \lambda$ is an even integer. If $Q_k \neq 0$, then $J_{k,1}$ is uniformly distributed on the interval $[P_k - |Q_k|, P_k + |Q_k|]$. This interval can be divided into $Q_k/2$ subintervals of the form $[\lambda - 1, \lambda + 1]$, where $N_e - \lambda$ is odd, and $J_{k,1}$ must be uniformly distributed on each of these subintervals. Thus, the proposition is true for Q_k even.

Now suppose that Q_k is odd. This implies that P_k is odd whenever N_e is even, and that P_k is even whenever N_e is odd. In any case, $N_e - P_k$ will be an odd integer. Q_k can never equal 0, so $J_{k,1}$ is uniformly distributed on the interval $[P_k - |Q_k|, P_k + |Q_k|]$. This interval can be divided into $(Q_k + 1)/2$ subintervals of the form $[\lambda - 1, \lambda + 1]$, where $N_e - \lambda$ is odd, and $J_{k,1}$ must be uniformly distributed on each of these subintervals. This completes the proof of the proposition.

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