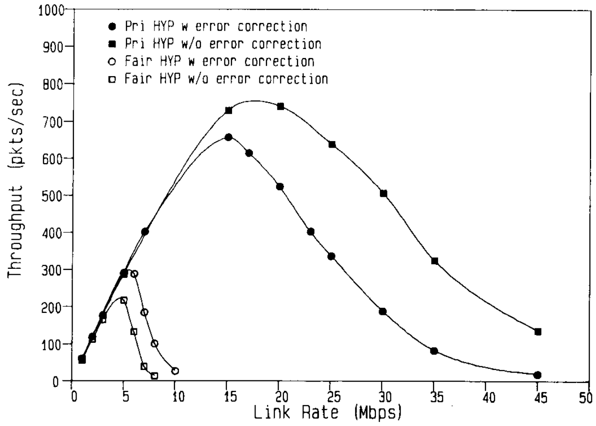

Fig. 8. Effects of eliminating LAN error correction: hyperchannel bus access protocols: $\lambda = 3.5, p = 6$.

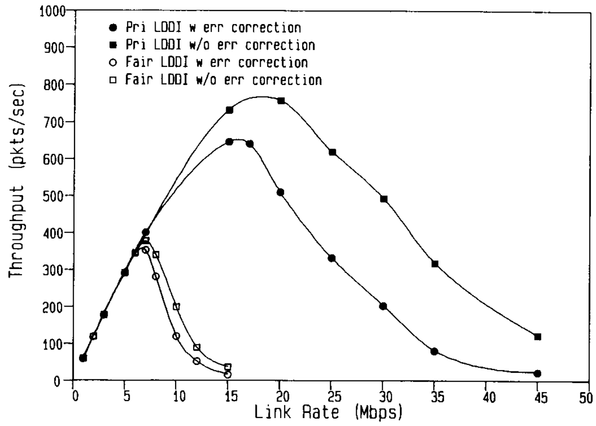

Fig. 9. Effects of eliminating LAN error correction: LDDI bus access protocols: $\lambda = 3.5, p = 6$.

improvement with $p = 1$, but is greater for lower gateway priorities.

On the other hand, eliminating LAN error correction can have an adverse effect on throughput as lost or damaged gateway packets cannot be retransmitted by the gateway and, consequently, the entire internet message is lost. Our simulation experiments indicate that collisions rarely occur when prioritized HYPERchannel is used, and eliminating error correction does not adversely affect throughput when using the prioritized HYPERchannel protocol. The detrimental effect of collisions on throughput when fair HYPERchannel is used can be observed in Figs. 7 and 8. With the fair HYPERchannel protocol gateway packets are frequently lost to collisions and, consequently, LAN error correction results in greater throughput than results from its omission. The fair LDDI protocol which avoids collisions and their adverse effects achieves higher throughput levels without LAN error correction than with error correction, as Fig. 9 demonstrates.

Our simulation results have demonstrated the performance characteristics of an interconnection network employing gateway adapters, and have been especially useful in understanding the effects of congestion on throughput as determined by local network traffic levels, local network bus access mechanism, gateway bus access priority, gateway buffering capacity, and use of local network error correction.

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Asymptotic Performance of *M*-ary Orthogonal Signals in Worst Case Partial-Band Interference and Rayleigh Fading

M. V. HEGDE AND W. E. STARK

***Abstract*—In this paper, we show that for large M the symbol error probability P_e of an orthogonal signal set transmitted over a channel with partial-band Gaussian interference is**

$$P_e = \lim_{M\to\infty} P_e(M) = \begin{cases} 1 & E_b/N_I < \ln 2 \\ \dfrac{\ln 2}{E_b/N_I} & E_b/N_I > \ln 2 \end{cases}$$

where E_b is the transmitted bit energy and N_I is the average power spectral density of the interference. This is in contrast to the additive white Gaussian noise channel which has asymptotic probability of error going to zero for $E_b/N_0 > \ln 2$. We also show that for a Rayleigh fading channel for large M the symbol error probability is $P_e = 1 - e^{-\ln 2/(E_b/N_0)}$. Finally, we provide numerical computations of the minimum E_b/N_0 required to achieve a symbol error probability of 10^{-5} to illustrate the asymptotic behavior described above.

I. Introduction

It is well known that when communicating over a white Gaussian noise channel an M-ary orthogonal signal set achieves capacity in that provided $E_b/N_0 > \ln 2$ the error probability can be made arbitrarily small [1]. It is also known [2] that provided codes of small enough rates are used the signal-to-noise ratio needed for reliable communication in the presence of a partial-band interferer is the same as that of

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white Gaussian noise. Thus, one might conjecture that M orthogonal signals would also provide reliable communication in the presence of a partial-band interferer. In this paper, we show that for worst case partial-band Gaussian jamming the error probability approaches $\ln 2/(E_b/N_I)$ for $E_b/N_I > \ln 2$ and thus M-ary orthogonal signals do not achieve capacity. For partial-band interference the error probability for $M = 2$, 4, 8, 16, and 32 was determined by Houston [3]. (The numerical results in Houston are slightly off for $M = 16$ and $M = 32$.) In [4], Crepeau and McGregor noted that while the exact bit error probability (from Houston's results) decreases with M (for $M = 2$, 4, 8, 16, and 32), the union bound increased with M. We will show the exact bit error probability is a decreasing function of M for small M ($M = 2, 4, 8, 16, 32, 64$) but then increases for large M ($M = 128, 256, \cdots$) although very slowly. The signal-to-noise ratio required for symbol error probability of 10^{-5} decreases for M less than 64 and increases thereafter.

For comparison purposes, we also derive the asymptotic form of the error probability for a Rayleigh faded channel. This was calculated previously by Lindsey [5] and approximated (for large signal-to-noise ratios) by Sussman [6]. For large signal-to-noise ratio (> 10 dB) this is well approximated by $\ln 2/(E_b/N_0)$ which is the same asymptotic limit as for the partial-band Gaussian jamming case. However, numerical examination indicates that the error probability is higher for the fading cases than for the partial-band cases for finite M.

II. Partial-Band Gaussian Interference

The effect of a partial-band Gaussian interference is to add Gaussian noise in *each* of the M dimensions with probability ρ and no noise with probability $1 - \rho$. Let E denote the energy of the transmitted signal and $E_b = E/\log_2 M$. Let N_I denote the equivalent one-sided white noise power spectral density. When jamming noise is present the noise power spectral density is N_I/ρ. Let $P_e(E_b/N_I, M)$ be the symbol error probability of an M-ary orthogonal signal set on an additive white Gaussian noise channel. Since the results in this paper apply equally well in both the coherent and noncoherent detection cases, we will allow $P_e(E_b/N_I, M)$ to indicate either case. For coherent detection [1]

$$P_e\left(\frac{E_b}{N_I}, M\right) = 1 - \int_{-\infty}^{\infty} (\Phi(u))^{M-1} \frac{1}{\sqrt{2\pi}} \cdot \exp\left\{\frac{-\left(u - \sqrt{\frac{2E_b}{N_I}\log_2 M}\right)^2}{2}\right\} du \tag{1}$$

where $\Phi(\cdot)$ is the distribution function of the standard normal distribution and for noncoherent detection

$$P_e\left(\frac{E_b}{N_I}, M\right) = 1 - \int_0^{\infty} u \exp\left\{-\frac{u^2 + \frac{2E_b}{N_I}\log_2 M}{2}\right\} \cdot I_0\left(\sqrt{\frac{2E_b}{N_I}\log_2 M}\, u\right)\left\{1 - \exp\left(-\frac{u^2}{2}\right)\right\}^{M-1} du$$

where $I_0(\cdot)$ is the modified Bessel function of the zeroth order. It is well known that [1]

$$\lim_{M\to\infty} P_e\left(\frac{E_b}{N_I}, M\right) = \begin{cases} 1 & E_b/N_I < \ln 2 \\ 0 & E_b/N_I > \ln 2 \end{cases} \tag{3}$$

in either the coherent or the noncoherent case. The error probability for a partial-band interference channel is then

$$P_{e,J}\left(\frac{E_b}{N_I}, \rho, M\right) = \rho P_e\left(\frac{E_b\rho}{N_I}, M\right). \tag{4}$$

The main result is

$$\text{Proposition: } \lim_{M\to\infty} \max_{0\le\rho\le 1} \rho P_e\left(\frac{E_b}{N_I}\rho, M\right) = \begin{cases} 1, & E_b/N_I < \ln 2 \\ \dfrac{\ln 2}{E_b/N_I}, & E_b/N_I > \ln 2. \end{cases}$$

The proof of this proposition is given in the Appendix. The result is perhaps a little surprising. Crepeau and McGregor [4] noted (from Houston's work) that the bit error probability decreased with M for $M < 32$. As mentioned earlier there are some (small) errors in Houston's numerical calculation of the constants that determine the error probability. Numerical examination of the bit error probability $\bar{P}_{e,b}(E_b/N_I, M) \triangleq M/2(M-1) \max_{0\le\rho\le 1} \rho P_e(E_b/N_I\rho, M)$ shows that in fact this function (for fixed E_b/N_I) is an increasing function of M (for $M \ge 64$). We have also found that the rate of convergence of $\bar{P}_{e,b}(E_b/N_I, M)$ to be extremely slow. The symbol error probability decreases for $M = 2$, 4, 8, 16, but then increases thereafter. Table I shows the value of E_b/N_I that achieves $\bar{P}_e(E_b/N_I, M) = 10^{-5}$ for several values of M. (Note this is the symbol error probability not the bit error probability.)

TABLE I
Signal-to-Noise Ratio Required to Achieve $P_e = 10^{-5}$

	Coherent Partial-Band Interference	Coherent Fading	Non-Coherent Partial-Band Interference	Non-Coherent Fading
	E_b/N_I	E_b/N_0	E_b/N_I	E_b/N_0
M=2	42.19	46.98	45.68	50.00
4	42.54	47.09	45.43	49.62
8	42.85	47.19	45.34	49.37
16	43.13	47.29	45.31	49.19
32	43.37	47.37	45.32	49.06
64	43.59	47.44	45.35	48.97
128	43.78	47.52	45.39	48.89
256	43.96	47.59	45.44	48.84
512	44.11	47.64	45.49	48.79
1024	44.25	47.69	45.54	48.76
2048	44.38	47.71	45.58	48.72
4096	44.50	47.72	45.63	48.70
⋮				
1068576	45.16	47.87	45.94	48.56
∞	48.41	48.41	48.41	48.41

III. Rayleigh Fading

For comparison purposes, we also consider the asymptotic performance for large values of M of a Rayleigh faded channel. These results have also been derived by Lindsey [5]. We are interested in nonselective fading. If R is a Rayleigh distributed random variable which represents the fading then the error probability, $P_{e,f}(E_b/N_0, M)$, for Rayleigh fading can be expressed as

$$P_{e,f}\left(\frac{E_b}{N_0}, M\right) = E\left[P_e\left(\frac{E_b R^2}{N_0}, M\right)\right] \tag{5}$$

where $P_e(E_b/N_0, M)$ is the error probability of M orthogonal signals in additive white Gaussian noise with power spectral density $N_0/2$ (again, we treat both the coherent and noncoherent cases together). To determine the limiting performance for large M, we apply the dominated convergence theorem

$$\lim_{M\to\infty} P_{e,f}\left(\frac{E_b}{N_0}, M\right) = \lim_{M\to\infty} EP_e\left(\frac{E_bR^2}{N_0}, M\right)$$
$$= E \lim_{M\to\infty} P_e\left(\frac{E_bR^2}{N_0}, M\right),$$

$$\lim_{M\to\infty} P_e\left(\frac{E_bR^2}{N_0}, M\right) = \begin{cases} 1, & \dfrac{E_bR^2}{N_0} < \ln 2 \\ 0, & \dfrac{E_bR^2}{N_0} > \ln 2 \end{cases}$$
$$= I(R < \sqrt{\ln 2/(E_b/N_0)})$$

where $I(R < \alpha)$ is the indicator function. Thus,

$$\lim_{M\to\infty} P_{e,f}\left(\frac{E_b}{N_0}, M\right) = \int_0^{\sqrt{\ln 2/(E_b/N_0)}} 2re^{-r^2/2}\, dr$$
$$= 1 - e^{-\ln 2/(E_b/N_0)}.$$

For large signal-to-noise ratio an excellent approximation is obtained by using $e^{-x} \approx (1 - x)$ for x small. Thus, for large E_b/N_0

$$\lim_{M\to\infty} P_{e,f}\left(\frac{E_b}{N_0}, M\right) \approx \frac{\ln 2}{(E_b/N_0)} \tag{8}$$

which is the same as that shown earlier for partial-band jamming. In Table I, we give numerical results for various values of E_b/N_0 and M.

IV. Conclusion

For worst case partial-band jamming the error probability performance (for fixed E_b/N_I) becomes worse with increasing M (for $M > 16$). The asymptotic probability of error is not zero for any E_b/N_I ($> \ln 2$) but decreases inverse linearly with respect to it. In the fading case, the error probability performance (for fixed E_b/N_0) improves with M for noncoherent detection but worsens with M for coherent detection. For large E_b/N_0 the performance of the Rayleigh fading channel asymptotically approaches the same limit as the worst case partial-band jammed channel. However, for values of M at least up to 4096, the partial-band jammed channel does better. While it it unlikely that a M-ary orthogonal signal set with $M > 1024$ will be used in a practical situation these results suggest an important theoretical problem, namely, what signal set achieves reliable communication.

Appendix A

Proposition:

$$\lim_{M\to\infty} \max_{0\le\rho\le 1} \rho P_e\left(\frac{E_b}{N_I}\rho, M\right) = \begin{cases} 1, & E_b/N_I < \ln 2 \\ \dfrac{\ln 2}{E_b/N_I}, & E_b/N_I > \ln 2. \end{cases}$$

Proof: To prove the proposition, we obtain an upper bound on

$$\limsup_{M\to\infty} \max_{0\le\rho\le 1} \rho P_e\left(\frac{E_b}{N_I}\rho, M\right)$$

and then an identical lower bound on

$$\liminf_{M\to\infty} \max_{0\le\rho\le 1} \rho P_e\left(\frac{E_b\rho}{N_I}, M\right).$$

This will prove the limit exists and is equal to the bounds. Clearly, $\max_{0\le\rho\le 1} \rho P_e(E_b/N_I, M) \ge P_e(E_b/N_I, M)$ so that for $E_b/N_I < \ln 2$,

$$\liminf_{M\to\infty} \max_{0\le\rho\le 1} \rho P_e\left(\frac{E_b}{N_I}\rho, M\right) \ge \liminf_{M\to\infty} P_e\left(\frac{E_b}{N_I}, M\right) = 1.$$

For $E_b/N_I > \ln 2$ and any $\epsilon > 0$

$$\max_{0\le\rho\le 1} \rho P_e\left(\frac{E_b\rho}{N_I}, M\right) \ge \left(\frac{\ln 2 - \epsilon}{E_b/N_I}\right) P_e(\ln 2 - \epsilon, M).$$

Thus,

$$\liminf_{M\to\infty} \max_{0\le\rho\le 1} \rho P_e\left(\frac{E_b\rho}{N_I}, M\right) \ge \liminf_{M\to\infty} \frac{\ln 2 - \epsilon}{E_b/N_I} \cdot P_e(\ln 2 - \epsilon, M) = \frac{\ln 2 - \epsilon}{E_b/N_I}$$

for $E_b/N_I > \ln 2$. Since this is true for any $\epsilon > 0$ we have proved that

$$\liminf_{M\to\infty} \max_{0\le\rho\le 1} \rho P_e\left(\frac{E_b\rho}{N_I}, M\right) \ge \begin{cases} 1, & E_b/N_I < \ln 2 \\ \dfrac{\ln 2}{E_b/N_I}, & E_b/N_I > \ln 2. \end{cases}$$

To obtain the upperbound, we first observe that $\rho P_e(E_b/N_I\, \rho, M) \le 1$ for $0 \le \rho \le 1$. Now, let ρ_M be the value of ρ that achieves the maximum in

$$\max_{0\le\rho\le 1} \rho P_e\left(\frac{E_b}{N_I}\rho, M\right).$$

Then for sufficiently large M, $\rho_M \le \ln 2/E_b/N_I \triangleq \gamma$. If this were not the case then for every M there would be an $M_1 > M$ and $\rho_{M_1} > \gamma$ for which

$$\rho_{M_1} P_e\left(\frac{E_b}{N_I}\rho_{M_1}, M_1\right) \ge \rho P_e\left(\frac{E_b}{N_I}\rho, M_1\right) \quad \text{for all } \rho,\ 0\le\rho\le 1. \tag{A.1}$$

For any $\epsilon > 0$ the right-hand side of (A.1) for sufficiently large M ($M > M_2$) can be made greater than $\gamma - \epsilon$. Since the left-hand side of (A.1) is obviously approaching 0 for $\rho_{M_1} > \gamma$, we have a contradiction. Thus, for sufficiently large M and $E_b/N_I > \ln 2$ $\max_{0\le\rho\le 1} \rho P_e(E_b/N_I\, \rho, M) = \max_{0\le\rho\le\gamma} \rho P_e(E_b\rho/N_I, M) \le \gamma$. This then proves that

$$\limsup_{M\to\infty} \max_{0\le\rho\le 1} \rho P_e\left(\frac{E_b\rho}{N_I}, M\right) \le \begin{cases} 1, & E_b/N_I < \ln 2 \\ \dfrac{\ln 2}{E_b/N_I}, & E_b/N_I > \ln 2 \end{cases}$$

and the proposition follows.

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