Interference Mitigation in Frequency-Hopped Spread-Spectrum Systems

Andrew Worthen and Wayne Stark [1]
Electrical Engineering and Computer Science
University of Michigan
Ann Arbor, MI 48105
e-mail: {worthena,stark}@eecs.umich.edu

Abstract — **In this paper we explore the error probability of different coding schemes in a frequency-hopped spread-spectrum communication systems subject to partial-band interference and a system subject to Rayleigh fading. The interplay between block length of the code and channel memory is quantified. We show that there is an optimal memory length that maximizes performance. At low signal-to-noise ratios (or close to capacity) large memory is better while at large signal-to-noise ratio smaller memory is optimum.**

I. Introduction

The performance of a coded frequency-hopped spread-spectrum communication system in the presence of interference with memory is considered in this paper. In particular we consider a code that is amenable to iterative decoding such as a turbo code or a low density parity check (LDPC) code. The goal is to investigate the performance of these codes on a frequency hopped channel that exhibits memory. If we consider a code with block length n and a frequency hopped system with m bits per hop then there are $L = n/m$ hops per codeword. A typical channel model for such a situation assumes the interference during each hop is constant. In the case of multipath fading this means that the fade level is a constant for each hop but independent from one hop to the next hop. For the case of other user interference it means that the interference is present for the whole hop so that the noise level is a constant for the whole hop. In this paper we consider the effect of channel memory on the performance of such a system for a fixed overall block length code. For turbo and LDPC codes, longer codes yield better the bit error probabilities (unlike convolutional codes) when used on a memoryless channel. In our scenario we have a fixed block length n code but the number of independent hops varies as the number of bits per hop changes. Thus if the number of bits per hop m is decreased then the number of independent hops increases and allows for more averaging of the noise/interference statistics. However, there are two other issues affecting performance. The first issue is the channel estimation scheme. If the receiver is able to accurately estimate the channel and use this information then the decoder can do a better job decoding than if no or only a poor channel estimate is available to the decoder. A better estimate of the channel during a given hop is possible if the length of each hop is large. Thus a small number of bits per hop decreases the capability of the receiver to make accurate estimates of the channel. Secondly, in order to synchronize to various signal parameters (e.g. phase, frequency, timing) a certain number of pilot symbols are sent on each hop. If the hop contains many bits per hop then the overhead for the pilot bits is small. However, if the hop contains only a few bits per hop then the overhead becomes a large percentage of the transmission time and the throughput can be significantly degraded.

In this paper we approach this problem in two ways. The first is from a random coding point of view. In this case we evaluate the error exponent as a function of the number of bits per hop but for a fixed total block length. For this approach we consider just the simple case of hard decision decoding. The second approach is to evaluate the performance of soft decision iterative decoding algorithms for LDPC codes incorporating channel estimation into the decoding algorithm. The results show that, without incorporating the overhead due to synchronization bits, the optimal number of bits per hop is 1 bit/hop at high SNR but at low SNR a larger number of bits per hop is better. So for large SNR the benefits of improved channel estimation do not overcome the improvement in performance due to more independent hops. Furthermore, as expected the difference between no side information available to the decoder about the channel and perfect information available about the channel disappears when the channel memory becomes large. When the overhead due to pilot symbols is incorporated into the performance evaluation the optimal number of bits per hops is strictly greater than one. As an example, we consider a block length 1024 code with 4 bits/hop for pilot bits. In this case the optimal number of bits per hop is about 30 bits. A smaller number of bits per hop causes excess overhead and poor channel estimation while an increase in the number of bits per hop decreases the effective block length of the code.

The remainder of the paper is organized as follows. In Section II we describe the channel models considered. In Section III we evaluate the reliability function for these channels with memory. In Section IV we present numerical results for the reliability function for random codes and the bit error probability for LDPC codes with a decoder that incorporates the channel estimation as part of the decoding. Finally, in Section V we present numerical results and conclusions.

II. System and Channel Model

A: Transmitter Model

In this section we describe the models for the systems and channel we consider. At the transmitter a data packet consist-

[1] This work was supported by DoD under and managed by ARO under the MURI program with grant number 96-1-0377. A. Worthen was supported by an NSF Graduate Research Fellowship.

ing of k bits of information with equal probability of taking on the values +1 and -1 is encoded into a codeword of length n coded bits. Each coded bit is used as the input to a BPSK modulator. The modulated signal is then frequency hopped over a set of nonoverlapping frequencies. The frequency hopping rate is such that m coded bits are sent over each hop. The transmitted signal is then given by

$$s(t) = \sqrt{2P} \sum_{l=0}^{n-1} x_l p_T(t - lT) \cos(2\pi f(t)t + \phi)$$

where x_l is the encoded bit sequence, $p_T(t)$ is a rectangular pulse shape of duration T starting at $t = 0$ and T is the encoded bit duration. The frequency pattern is given by

$$f(t) = \sum_{j=0}^{L-1} f_j p_{T_h}(t).$$

where $L = n/m$ and T_h is the hop duration. The transmitter sends the first m coded bits at frequency f_1 before hopping to a frequency f_2 where another m bits are transmitted and so on until the last m bits are transmitted at frequency $f_{(n/m)-1}$. In order to synchronize to the frequency, phase, and timing of the bits at each frequency a number of pilot symbols are typically transmitted during each hop. If the number of coded bits per hop is small then these pilot bits cause the overhead to be quite large. If p pilot bits and m coded bits are transmitted on each hop the $T_h = (m + p)T$.

B: Channel Model

In this paper we consider two different channel models. The first model is that of a interference channel where with probability ρ the noise level for a whole hop is $N_0/2 + N_J/(2\rho)$ and with probability $1 - \rho$ the noise level is $N_0/2$. Furthermore the random variables characterizing the noise level at a given hop $z_0, ..., z_{L-1}$ form a sequence of independent random variables. The second channel model is that of a slowly faded channel where the coherence time is much longer that the hop duration, and the coherence bandwidth is larger than the hop bandwidth but smaller than the separation between two hop frequencies. In this case the channel exhibits frequency nonselective fading over a hop but independent fading between one hop and the next. The fading within a single hop is constant.

C: Receiver

The receiver processes the received signal by first frequency dehopping the received signal and then demodulating the resulting signal. The demodulated outputs are denoted by $y_0, ..., y_{n-1}$. We assume perfect synchronization and timing is possible with the p pilot bits transmitted on each hop. The outputs of the demodulator corresponding to the encoded bits are as follows. For the case of an interference channel

$$y_l = \sqrt{E} x_l + \eta_l, \quad l = 0, 1, ..., N - 1$$

where E is the received energy per coded bit. The noise η_l is a sequence of Gaussian random variables conditioned on the interference level. For the j-th hop the noise is Gaussian with mean zero and variance $N_0/2 + z_j N_J/(2\rho)$ where z_j is one with probability ρ and is zero with probability $1 - \rho$.

For the case of a fading channel the input-output relation of the channel is given by

$$y_l = \sqrt{E} r_{\lfloor l/m \rfloor} x_l + \eta_l, \quad l = 0, 1, ..., N - 1.$$

In this case the noise is a sequence of independent identically distributed Gaussian random variables with mean zero and variance $N_0/2$. The variables r_l are Rayleigh distributed with second moment $2\sigma^2 = 1$ which makes E the average received energy per coded bit. The variable r_j corresponds to the fade level during the j-th hop. The fading on different hops is assumed independent and identically distributed.

Both of these models are special cases of a block interference model [1]. The analysis for block interference channels is straightforward after realizing that the channel can be viewed as a memoryless channel on a hop by hop basis. This realization makes calculation of the channel capacity straightforward.

III. Performance Analysis

The fundamental limits for channels of the type described above have been determined previously [1]. Here we review these results and then discuss the reliability function for these channels. Because of the difficulty in computing the reliability function for these channels we only consider the case where the receiver makes a hard decision regarding each coded bit at the receiver. That is we form the vector $(\hat{y}_0, \hat{y}_1, ..., \hat{y}_{n-1})$ where $\hat{y}_l = +1$ if $y_l > 0$ and is equal to -1 otherwise. In addition, we consider the two possibilities of channel information available to the receiver. If the receiver knows (by some genie) the channel state (interference level or fade level) for each hop then we say channel side information is available. Normally this must be estimated from the received signal. The larger the number of bits per hop the more accurate the estimate of the fade. The capacity without side information depends on the number of bits per hop (or memory). As the memory (number of bits/hop) increases, the capacity increases and approaches the capacity with perfect side information. Interestingly, the cutoff rate has the opposite behavior. That is, the cutoff rate decreases as the channel memory increases. This result has been used to claim that interleaving is a good idea [2]. So given these two opposite conclusions about performance how are these conflicting results properly explained and interpreted? To explain these results we consider the reliability function of block interference channels. The reliability function, as will be shown below, incorporates both the capacity and the cutoff rate. At low rates the reliability function is equal to the difference of the cutoff rate and the code rate while at high rates the reliability function remains positive below the capacity. From this it is clear that for two different memory lengths the reliability function is larger at small rates for smaller memory while at high rates the reliability is higher for larger memory.

To determine the reliability of a channel we define $x_h = (x_0, ..., x_{m-1})$ and $y_h = (y_0, ..., y_{m-1})$ to be vector of length m. The channel hop transition probability $p_h(y_h|x_h)$ for the interference channel is given by

$$\begin{aligned} p_h(y_h|x_h) &= p_h(y_h|x_h, z = 1)P\{z = 1\} \\ &\quad + p_h(y_h|x_h, z = 0)P\{z = 0\} \\ &= \frac{\rho}{\sqrt{2\pi}\sigma_J} \exp\{-\frac{1}{\sigma_J} \sum_{l=0}^{m} (y_l - \sqrt{E} x_l)^2\} \end{aligned}$$

$$+\frac{1-\rho}{\sqrt{2\pi}\sigma_0}\exp\{-\frac{1}{\sigma_0}\sum_{l=0}^{m}(y_l-\sqrt{E}x_l)^2\}$$

where $\sigma_J^2 = (N_0 + N_J/\rho)/2$ and $\sigma_0^2 = N_0/2$. The channel hop transition probability for the case of the Rayleigh faded channel is given by

$$\begin{aligned} p_h(y_h|x_h) &= \int_0^\infty p_h(y_h|x_h, r) f(r) dr \\ &= \int_{r=0}^\infty p_h(y_h|x_h, r)\frac{r}{\sigma^2}\exp\{-r^2/2\sigma^2\}dr \end{aligned}$$

where

$$p_h(y_h|x_h, r) = \frac{1}{\sqrt{\pi N_0}}\exp\{-\frac{1}{N_0}\sum_{l=0}^{m}(y_l-\sqrt{E}rx_l)^2.$$

These determine the input output of the channel. Note that the transition probability does not factor (except if $\rho = 1$) into the product of transition probabilities for each element of the vector.

The reliability of a memoryless channel with input x_h and output y_h is given by [3]

$$\begin{aligned} E_m(R) &= \max_{0\le s\le 1}\max_{p_h(x_h)}[sR \\ &- \log_2\int_{y_h}\left(\sum_{x_h}p_h(x_h)[p_h(y_h|x_h)]^{\frac{1}{1+s}}\right)^{1+s}\Bigg]. \end{aligned}$$

Unfortunately, the reliability function is difficult to calculate since it involves integration over an m dimensional space. However we can efficiently calculate the reliability when the demodulator makes hard decisions about each code bit. In this case the channel transition probabilities are determined by (for the fading case)

$$p_h(\hat{y}_h|x_h) = \int_0^\infty\left[Q^l(\sqrt{\frac{2Er}{N_0}})(1-Q(\sqrt{\frac{2Er}{N_0}}))^{m-l}\right]f(r)dr$$

where $d_h(\hat{y}_h, x_h) = l$ is the Hamming distance between the (binary) input and the output. For the case of partial-band interference

$$p_h(\hat{y}_h|x_h) = \rho P_J^l(1-P_J)^{(m-l)} + (1-\rho)P_0^l(1-P_0)^{(m-l)}$$

where

$$P_J = Q\left(\sqrt{\frac{2E}{(N_0+N_J/\rho)}}\right)$$

and

$$P_0 = Q\left(\sqrt{\frac{2E}{N_0}}\right).$$

We also consider the reliability function with side information about the channel state (fade level or noise level). In this case we treat the output as not only y_h (or $\hat{y}_h$) but as the combination of y_h with r_j or z_j where r_j represents the fade level for a hop and z_j represents the presence of a jammer during a hop. Thus we replace $p_h(y_h|x_h)$ with $p_h(y_h, r|x_h)$ for the case of fading and we replace $p_h(y_h|x_h)$ with $p_h(y_h, z|x_h)$ for the case of interference. These can be easily determined

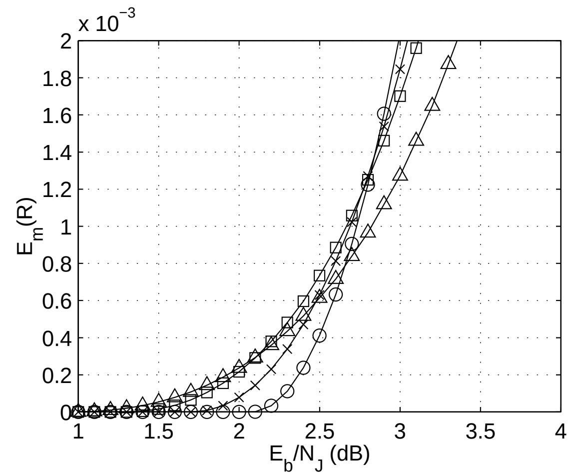

Figure 1: Reliability function for rate 1/2 codes and memory $m = 1, 16, 32, 64$ and $\rho = 0.7$.

from the above equations. The reliability function is related to the error probability for random codes via

$$P_e \le 2^{-NE_m(R)} \tag{1}$$

For many channels (both of the ones described above) the best input distribution is the uniform, iid distribution which is what we assume subsequently. At low rates (below R_{crit}) it is known that the reliability function is

$$E(R) = R_0 - R$$

where

$$\begin{aligned} R_0 &= \log_2\sum_{y_h}\left(\sum_{x_h}p_h(x_h)[p_h(y_h|x_h)]^{\frac{1}{2}}\right)^2 \\ R_0 &= \log_2\sum_{y_h}\sqrt{p_h(y_h|x_h)p_h(y_h|x_h')} \end{aligned}$$

where x_h and x_h' are two distinct input vectors. It is also known that the reliability function is nonzero at rates up to the channel capacity.

IV. Results

We first consider the case of partial-band interference and examine the reliability function for this channel. The reliability function is related via (1) to a bound on the error probability for random codes. It is known that there exist codes and a decoding algorithm with error probability less than the right hand side of (1). In all the results that follow the signal-to-background noise E_b/N_0 is fixed at 10dB. Note also that the energy per information bit E_b is related to the energy per code bit E via $E_b = E/R$. For the results with partial-band interference we assume there are no pilot bits.

In Figure 1 we plot the reliability function for a partial-band interference channel for different memory lengths as a function of the signal-to-noise ratio. As can be seen from the figure for small signal-to-noise ratios (closer to capacity) the reliability function is largest for the larger memory channels while for small signal-to-noise ratios the reliability is largest for small memory channels. The interpretation is that for large

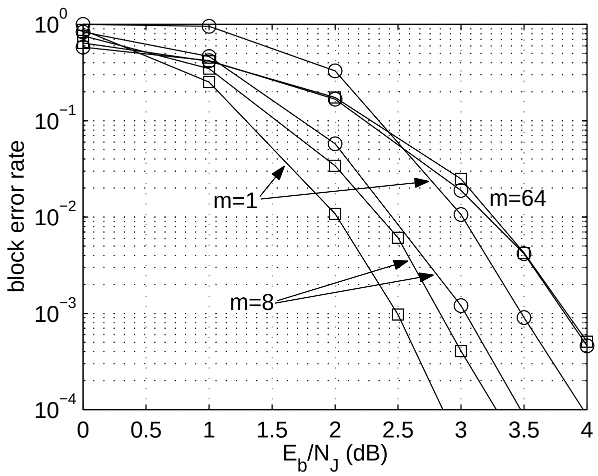

Figure 2: Error probability for for LDPC codes with rate 1/2 codes and memory $m = 1, 8, 64$ and $\rho = 0.7$ ($\square$ = side information avaiable, o= no side information).

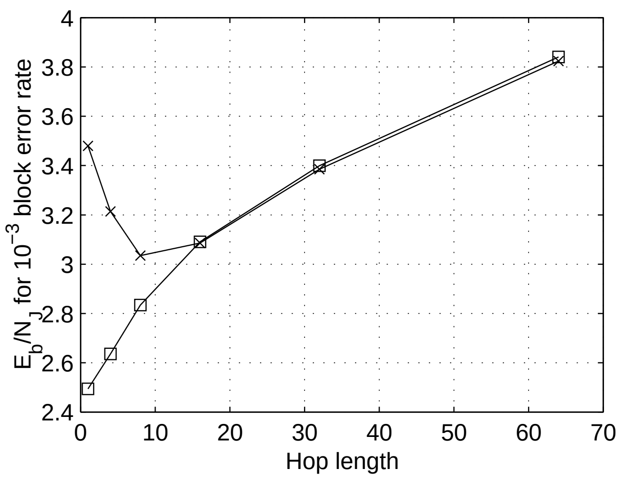

Figure 3: Signal-to-noise ratio require for block error probability 10^{-3} for rate 1/2 LDPC codes as a function of memory for $\rho = 0.7$.

signal-to-noise ratio the information about the channel state is not important while at low signal-to-noise ratios (close to capacity) the channel state is much more important and thus larger memory yields better channel estimation.

Now consider a low density parity check code for these channels. For the case of no state information available to the decoder we can design a decoder that attempts to approximate a maximum likelihood decoder. This decoder uses the information from the channel to do joint data and channel state estimation. The algorithm is based on representing the channel and the encoder with a factor graph and applying an iterative algorithm to approximated the maximum likelihood decoder. More details on this can be found in [4]. In Figure-fig:ldpcsnr we plot the block error probability for an LDPC code of length 1024 on a partial-band interference channel with 1,8 and 64 bits per hop. For the case where the receiver knows the channel state (jammed or unjammed) the best performance is obtained with memory 1 while for the case where the receiver does not know the channel state the best performance in the range of interest is obtained with memory 8 bits per hop. As the memory increases, the gap between the no side information case and the side information case decreases. The gap is about 1.2dB for memory 1 while for memory 64 the performance is virtually identical.

In Figure 3 we plot the signal-to-jamming noise required for a block error probability of 10^{-3} as a function of the number of bits per hop for a fixed block length of 1024. The top curve represents the situation where there is no side information available while the bottom curve represents the case of perfect side information. As can be seen from this figure for the case of no side information available there is an optimum number of bits/hop (m). If the memory is too small the performance degrades due to inaccurate state estimation while if the memory is too large the performance degrades due to the small number of independent hops per codeword.

Now consider the fading channel described in Section II. We begin by plotting the error probability from the random coding bound in (1). To keep comparisons fair we assume that $(p+m)L$ is the total number of bits available for transmission. There are pL pilot bits and $n = mL$ code bits. In Figure 4 the random coding error probability is plotted as a function of the average received signal-to-noise ratio for $(p+m)L = 1024$. We plot the performance with perfect side information assumed available at the receiver about the state of the channel (fade level) and without side information available. We assume that there are $p = 4$ pilot bits in a hop along with m coded bits where m is either 12 or 124. Several observations can be drawn from the plot. First, it is clear that for $m = 124$ the difference between doing optimal demodulation without the side information does not degrade the performance compared to knowing the side information exactly (about 0.2dB) whereas for $m = 12$ the degradation is much more significant (about 2dB). This confirms the intuition that longer hop length leads to better estimation of the channel. Another observation is that at high signal-to-noise ratios the error probability for the $m = 12$ case is smaller than for the $m = 124$ case. This is because we are not operating the channel close to capacity. We know that if the rate is sufficiently smaller than capacity that the reliability function is larger for smaller memory. However as small signal-to-noise ratios (below 10dB in this example) the larger memory performs better than the smaller memory. We note that for $m = 128$ bits/hop there are only 8 hops with each having 4 pilots bits. In this case the actual code is a $n = 992$, $k = 512$ code while for $m = 16$ there are 64 hops and the code is a $n = 768$, $k = 512$ code. The total number of symbols used (pilot and coded bits) are the same in both cases. If fewer pilot symbols were present (or none) the $m = 12$ curve would shift more to the left (lower SNR) than the $m = 124$ curve and thus the crossover between smaller memory and larger memory would occur at a lower SNR. If the block length were larger then the curves would shift down by an equal amount which would shift the crossover point to a much lower block error probability.

Now consider some specific codes for these slow fading block interference channels. One obvious choice is interleaved Reed-Solomon codes. For example a block length 32 Reed-Solomon code with 32 interleaved codewords. In this case since the alphabet size is 2^5 a single hop consists of 32 code symbols each of consisting of 5 bits. The Reed-Solomon code has rate 1/2 so there are 16 information symbols per codeword. Overall,

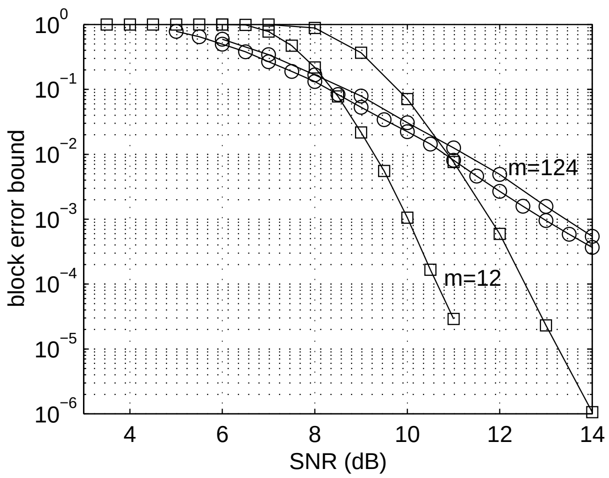

Figure 4: Random coding bound on error probability for memory 16 and 128

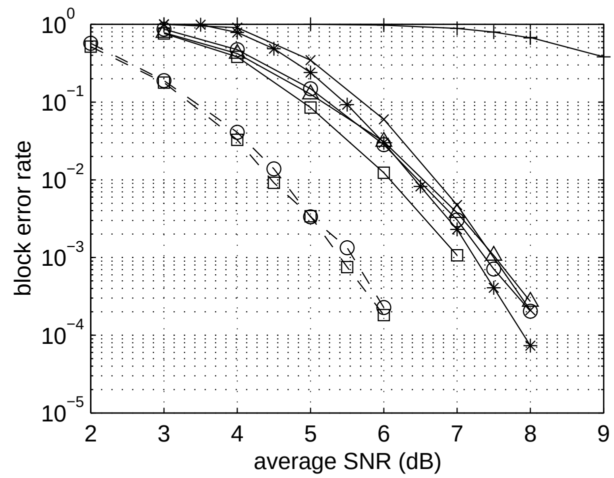

Figure 5: Error probability for random, Reed-Solomon and LDPC codes on frequency hopped channel with 160 bits/hop

there are $n = 5120$ coded bits. In Figure 5 the performance of various coding schemes are plotted. The top curve in this figure is the performance of Reed-Solomon codes with hard decision decoding. The next curves (with *'s and x's) represent the performance of random codes (from the random coding bound) with and without side information about the channel state. The curve with triangles represent the performance of Reed Solomon codes with a decoder that has perfect information about whether each symbol was in error or not. Any other decoder which tries to estimate the performance will clearly do worse than a decoder that has perfect information about which symbols were in error and erases those symbols. Finally we plot the performance of a low density parity check code with the same overall block length and no interleaving (circles and boxes). For the decoder we plot (in boxes) the cases of the receiver having perfect side information about the fade level (and thus the error probability in the case of hard decision decoding) as well as a decoder that tries to jointly estimate the channel state as well as the data similar in nature to that in [4]. The decoder assumes a six state model for the fade level and assigns an a priori distribution to the states similar to that of a Rayleigh distributed random variable.

Reed-Solomon codes with hard decision decoding and no side information available require an average received signal-to-noise ratio of about 12.5dB which is about 5.5dB worse than the Reed-Solomon decoder with perfect information about the occurrence of errors. From this figure it is clear that low density parity check codes without any channel information performs comparably to the Reed-Solomon code with perfect information yet does not require any interleaving. In addition we plot the performance of the low density parity check code with soft decision decoding with and without side information. As can be seen the case where no side information is present has essentially the same performance as the case with side information. Further more the soft decision decoding algorithms is the usual 2dB better than the hard decision decoding algorithm.

V. Conclusions

In this paper we have examined the performance of a frequency-hopped spread-spectrum system in two different interference environments: self interference from multipath fading and partial-band interference. We quantified the performance for random codes via the reliability function in the case of hard decision decoding. We saw the existence of an optimal amount of memory (hop length) for the objective function of block error probability. (Similar results would hold for bit error probability). We also demonstrated this interesting behavior for low density parity check codes. These results apply equally well to TDMA systems where the hop length is reinterpreted to be the burst length. We hope these results finally put to rest all myths regarding how to deal with memory in communication system design.

References

[1] R. J. McEliece and W. E. Stark, "Channels with block interference," *IEEE Transactions on Information Theory*, vol. IT-30, pp. 44–53, January 1984.

[2] A. J. Viterbi, "Spread-spectrum communications—myths and realities", *IEEE Communications Magazine*, 4:11–18, May 1979.

[3] R. E. Blahut, *Principles and Practice of information Theory*, Addison-Wesley, Reading, MA, 1987.

[4] A. P. Worthen and W. E. Stark, "Low-density parity check codes for fading channels with memory," in *Proceedings of the 36th Annual Allerton Conference on Communications, Control, and Computing*, pp. 117–125, September 1998.