Performance of Robust Metrics with Convolutional Coding and Diversity in FHSS Systems under Partial-Band Noise Jamming

Kyungwhoon Cheun and Wayne E. Stark, *Member, IEEE*

Abstract—**Performance of robust metrics (metrics that can be computed from the outputs of the matched filters only) with convolutional coding and diversity under worst case partial-band noise jamming is analyzed. Both binary and dual-k convolutional codes employing these metrics with diversity are compared via Union–Chernoff bounds. The performances of metrics considered in the literature that assume perfect side-information are given for comparison purposes. We find that there exists very good robust metrics that provide performance comparable to metrics using perfect side-information. Among the robust metrics considered in this paper, the self-normalized metric offers the best performance and achieves performance practically identical to that of the square-law-combining metric with perfect side-information for M = 8.**

I. Introduction

It is well known that diversity (i.e., transmission of a code symbol over many different hops), concatenated with a forward error-correcting code is an effective means of thwarting the partial-band noise jammer in a frequency-hop spread-spectrum (FHSS) communications system. There has been considerable investigation aimed at finding effective methods of combining the diversity transmissions [1]–[8].

When block coding with algebraic decoders are employed, usually, the diversity transmissions must first be combined and decisions made on each of the received symbols before a received vector can be decoded. On the other hand, when convolutional codes with Viterbi decoding are employed, hard decisions on the code symbols need not be made since the Viterbi decoder is very well suited for soft decision decoding. In this paper, we consider convolutional coding with L diversity transmissions for each code symbol. We assume that a Viterbi decoder corresponding to the employed convolutional code with each symbol repeated L times is used unless the diversity transmissions are first combined and a decision is made on each convolutional code symbol (e.g., metric (2) described below.) We assign a metric to each received diversity symbol and the Viterbi decoder chooses the path with the largest or smallest metric as the surviving path for each state depending on the metric used.

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K. Cheun was with the Department of Electrical Engineering and Computer Science, University of Michigan, Ann Arbor, MI 48109. He is now with the Department of Electrical Engineering, Pohang Institute of Technology, Pohang, Korea.

W. Stark is with the Department of Electrical Engineering and Computer Science, University of Michigan, Ann Arbor, MI 48109.

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Among many possible metrics, we focus on the metrics that can be computed from the outputs of the matched filters only and does not require the knowledge of any other system parameters such as the signal-to-noise ratio or the exact knowledge of the state of the channel (perfect side-information.) Such metrics will be referred to as *robust* metrics. Robust metrics are useful in spread-spectrum communications systems where information other than that obtained from the matched filter outputs may be difficult to obtain. It is shown here that there exists good robust metrics with performance very close to that of square-law-combining metric with perfect side-information [6].

The following are the metrics that will be considered in this paper. Each will be referred to in the following discussions by their abbreviations given inside the parenthesis or by their number.

1) Hard decisions on each diversity transmission and Hamming distance metric. (Hard No SI).

2) Hard decisions on each diversity transmission and errors and erasure decision on the convolutional code symbols and Hamming distance metric. (Hard E&X No SI).

3) Viterbi ratio thresholding with effective errors and erasure decision for each diversity transmission and Hamming distance metric. (VRT E&X).

4) Viterbi ratio thresholding with effective M-ary to $2M$-ary channel for each diversity transmission and integer metric. (VRT Integer).

5) Self-normalized metric. (Self-Normalized).

6) Square-law-combining metric with perfect side-information. (Soft SI).

7) Hard decisions on each diversity transmission with perfect side-information and Hamming distance metric. (Hard SI).

Metrics (1), (6), and (7) are more or less standard metrics considered here for comparison purposes whereas metrics (6) and (7) assume perfect side-information. Coded performance of metrics (1) and (6) were also considered in [9]. Metric (4) for a FHSS system with partial-band noise jamming was first considered in [10] where information theoretic analysis was done to confirm the robustness of the thresholding technique. Also given in [10] are the error probability versus

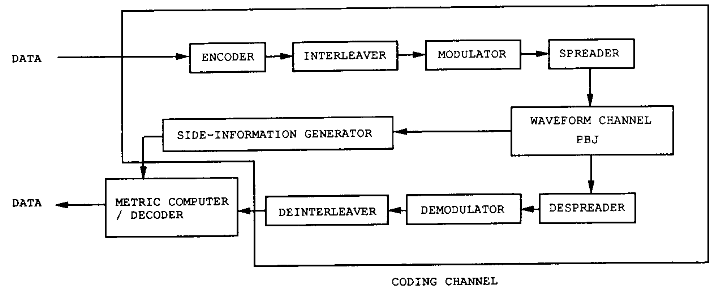

Fig. 1. Frequency-hop communications system block diagram.

signal-to-noise ratio plots for binary convolutional codes with M-ary frequency shift keying (MFSK) with the fraction of the spread-spectrum band jammed, ρ as a parameter. Exact error probability analysis of metric (5) for binary frequency shift keying (BFSK) and diversity was carried out in [3]. For the case when MFSK ($M > 2$) is employed, it was demonstrated in [11] that evaluation of the exact expression for the probability of error is extremely tedious even for $M = 4$ and diversity $L = 2$. In this paper, we resort to the Chernoff bound on the error probability and present numerical results up to $M = 8$. It is shown that for $M = 8$, the self-normalized metric offers performance practically identical to that of the square-law-combining metric with perfect side-information.

The general block diagram of the communications system under consideration is shown in Fig. 1. The modulation scheme we consider is orthogonal frequency shift keying with noncoherent demodulation. For BFSK, we consider binary convolutional codes while for MFSK ($M > 2$), we consider both binary and dual-k convolutional codes [12]. Binary convolutional codes with MFSK ($M > 2$) modulation is used by interleaving the $k = \log_2 M$ binary code symbols transmitted in an M-ary symbol in order to make the coding channel memoryless.

The jammer model we consider is that of an on-off partial-band noise jammer. We ignore the effect of the background thermal noise and assume that the system performance is dominated by the jammer. The (on-off) partial-band noise jammer spreads Gaussian noise of total power J uniformly over a bandwidth $W_J \le W_{ss}$ where W_{ss} is the total spread bandwidth used by the FHSS communications system. The fraction of the band jammed denoted by ρ is defined as, $\rho \triangleq W_J/W_{ss}$ where $0 < \rho \le 1$. Defining $N_J = J/W_{ss}$, the two-sided noise power spectral density in the band jammed is given by $\tilde{N}_J = N_J/\rho = J/W_J$. Thus, the signal-to-noise ratio when a particular hop is jammed is $E_c/\tilde{N}_J = \rho(E_c/N_J)$ where E_c/N_J is the signal-to-noise ratio for each hop for the broadband noise jammer ($\rho = 1$). Also, $E_c/N_J = (rk/L)(E_b/N_J)$ where E_b/N_J is the information bit signal-to-noise ratio for the broadband jammer, L is the diversity level and r is the rate of the convolutional code employed. We make the usual assumption that the channel is memoryless and stationary in the sense that the channel statistics are independent from hop to hop and fixed within a hop. Also a hop is assumed to be jammed in its entirety if it is jammed at all.

Performances of different metrics will be compared using Union–Chernoff bounds. This bound is not especially a tight bound but has the advantage of being relatively easy to compute and decouples the coding channel influence from the code itself. For each E_b/N_J there is a worst case ρ that maximizes the Union–Chernoff bound which we will denote by ρ^*. The performance measure we consider is the minimum bit signal-to-noise ratio E_b/N_J needed to guarantee the Union–Chernoff bound on the bit error probability (P_b) to be less than 10^{-5} for $\rho = \rho^*$ denoted by $E_b/N_J|_{10^{-5}}$. For some of the more interesting metrics we plot $E_b/N_J|_{10^{-5}}$ versus ρ and the bit error probability versus the bit signal-to-noise ratio.

In Section II, Chernoff parameters for the metrics considered in this paper are derived. In Section III, numerical results for various performance measures are given for the metrics and codes considered and, finally, conclusions are drawn in Section IV.

II. Union–Chernoff Bounds

The Union–Chernoff bound is widely used to upper bound error probabilities of convolutional codes decoded with a Viterbi decoder using various metrics. It is based on the Chernoff bound on the pairwise error probability, i.e., the probability that the output of the decoder is the code sequence $\hat{\boldsymbol{x}}$ when the sequence $\boldsymbol{x}$ was actually transmitted. This is given by

$$P(\boldsymbol{x} \to \hat{\boldsymbol{x}}) \le D^{W_H(\boldsymbol{x},\hat{\boldsymbol{x}})} \tag{1}$$

where $\boldsymbol{x}, \hat{\boldsymbol{x}}$ are two coded sequences, $W_H(\boldsymbol{x},\hat{\boldsymbol{x}})$ is the Hamming distance between $\boldsymbol{x}$ and $\hat{\boldsymbol{x}}$ and D is the Chernoff parameter defined by

$$D = \max_{0<\rho\le 1} D(\rho) \tag{2}$$

where $D(\rho)$ is given by [6]

$$D(\rho) = \min_{\lambda \ge 0} D(\rho, \lambda) \tag{3}$$

and

$$D(\rho, \lambda) = E\{\exp(\lambda(m(\boldsymbol{y}, \hat{x}; z) - m(\boldsymbol{y}, x; z))|x\}|_{x \ne \hat{x}}. \tag{4}$$

Here $m(\boldsymbol{y}, x; z)$ is the metric corresponding to symbol x given that $\boldsymbol{y}$ is the received vector and the side-information is z if any. The expectation E is over $\boldsymbol{y}$ and z. It is not hard to check that D does not depend on x or $\hat{x}$ for the metrics and modulation scheme considered in this paper as long as $x \neq \hat{x}$. For the special case of maximum likelihood metric, minimization over λ reduces $D(\rho)$ to the Union–Bhattacharyya bound given by [4]

$$D(\rho) = \oint_{\boldsymbol{y}} \sqrt{p(\boldsymbol{y}|x)\, p(\boldsymbol{y}|\hat{x})}|_{x \neq \hat{x}}\, d\boldsymbol{y} \tag{5}$$

where $\oint_{\boldsymbol{y}}$ denotes summation over $\boldsymbol{y}$ for discrete output channels and integration over $\boldsymbol{y}$ for continuous output channels. The Chernoff parameter D with diversity can be computed by first computing D for the diversity channel (i.e., channel for a single hop) and raising it to the power of L.

The Chernoff parameter D can be used with the weight distribution of the convolutional code to provide an upper bound on the bit error probability p_b of the convolutional codes as follows [13]

$$P_b \leq \frac{1}{b} \sum_{d=d_f}^{\infty} w_d D^d \tag{6}$$

where w_d is the number of input 1s in all the finite length coded sequence of Hamming weight d and b is the number of information bits in one branch of the trellis corresponding to the code. Also d_f denotes the minimum free distance of the code. For regions of interest ($P_b < 10^{-4}$), the infinite summation can be accurately approximated by a finite (5 or 6 terms) summation. For various optimal binary codes, the weight distribution $\{w_d\}$ can be found in [14]. Dual-k codes are nonbinary codes optimized for M-ary orthogonal modulation. The transfer function and hence the weight distributions for all optimal dual-k codes of rate $1/\nu$ are known [12] and the bit error probability can be bounded as

$$P_b \leq \frac{2^{k-2} D^{2\nu}}{(1 - \nu D^{\nu-1} - (2^k - 1 - \nu) D^{2\nu})^2}. \tag{7}$$

Next, we present the Chernoff parameters for the metrics listed in Section I.

A. Hard No SI

The first system we consider makes hard decisions on each of the diversity transmissions by deciding that the symbol corresponding to the matched filter with the largest output was sent, thus creating an effective M- ary symmetric channel shown in Fig. 2 for each hop. This system was considered by Levitt and Omura [2]. If the symbol error probability of this M-ary symmetric channel is less than $(M-1)/M$, the Viterbi decoder that minimizes the Hamming distance is a maximum likelihood decoder and the Chernoff bound reduces to the Bhattacharyya bound. The expression for the parameter D is given by [4]

$$D = \max_{0 < \rho \leq 1} \left[\sqrt{\frac{4P_S(1-P_s)}{M-1}} + \left(\frac{M-2}{M-1}\right) P_S \right]^L \tag{8}$$

where

$$P_S = \rho \sum_{l=1}^{M-1} \binom{M-1}{l} \frac{(-1)^{l+1}}{l+1} \exp\left(-\frac{l}{l+1} \rho \frac{E_c}{N_J}\right). \tag{9}$$

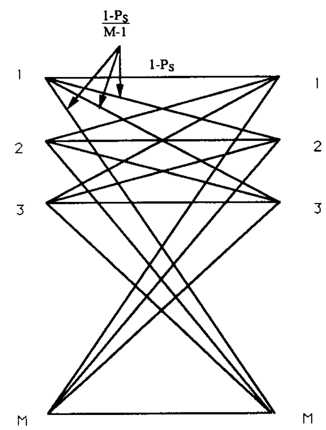

Fig. 2. M-ary symmetric channel.

For $M > 2$ and perfect interleaving of the binary convolutional code symbols, the resulting coding channel is a memoryless binary symmetric channel (BSC) with transition probability

$$p = \frac{M}{2(M-1)} P_S \tag{10}$$

and the Chernoff bound simplifies to

$$D = \max_{0 < p \leq 1} [2\sqrt{p(1-p)}]^L. \tag{11}$$

B. Hard E&X No SI

The next system we consider is similar to the system described above but here, errors and erasure decisions on each convolutional code symbol is made by observing the hard decisions made on the L diversity transmissions corresponding to the code symbol. We will only consider BFSK for this case. We use the following rule for erasures. Let N_1 denote the number of 1s in the L diversity receptions. Then for L even,

- erase if: $L/2 - T \leq N_1 \leq L/2 + T$,
- decide 1 sent if: $L/2 + T < N_1 \leq L$,
- decide 0 sent if: $0 \leq N_1 < L/2 - T$

where T is an integer threshold between 0 and $L/2$ that should be chosen appropriately. Let us denote the error and erasure probability for the resulting coding channel and the transition probability of the BSC that results from the hard decisions made on the diversity transmissions by p_e, p_x, and p, respectively. Then

$$p_x = \sum_{l=L/2-T}^{L/2+T} \binom{L}{l} p^l (1-p)^{L-l} \tag{12}$$

$$p_e = \sum_{l=L/2+T+1}^{L} \binom{L}{l} p^l (1-p)^{L-l}. \tag{13}$$

For L odd the rule is to

- erase if: $L - 1/2 - T < N_1 < L + 1/2 + T$,
- decide 1 sent if: $L + 1/2 + T \leq N_1 \leq L$,
- decide 0 sent if: $0 \leq N_1 \leq L - 1/2 - T$

where $0 < T \leq L - 1/2$. The erasure and error probabilities for this case are given by

$$p_x = \sum_{l=L-1/2-T+1}^{L+1/2+T-1} \binom{L}{l} p^l (1-p)^{L-l} \quad (14)$$

$$p_e = \sum_{l=L+1/2+T}^{L} \binom{L}{l} p^l (1-p)^{L-l}. \quad (15)$$

For this metric, the decoder that ignores the erased positions and minimizes the Hamming distance among the nonerased positions is the maximum likelihood decoder as long as the error probability is less than 1/2. Thus,

$$D = \max_{0<p\leq 1} [p_x + 2\sqrt{p_e p_c}] \quad (16)$$

where $p_c = 1 - p_e - p_x$.

C. VRT E&X

In this system Viterbi ratio thresholding is used to make errors and erasure decisions on the diversity transmissions. Let $Y_1, \cdots, Y_M$ denote the outputs of the matched filters (square-law detectors) corresponding to the M modulated signals, and let $Y_{\max}$ denote the maximum of $\{Y_1, \cdots, Y_M\}$ and let Y' denote the second largest. The rule is to erase the symbol if $1 \leq Y_{\max}/Y' < \theta$ and decide that the symbol corresponding to $Y_{\max}$ was sent if $Y_{\max}/Y' \geq \theta$ for some θ greater than 1 that should be set to an appropriate value. Analysis by Viterbi [10] shows that with an appropriate choice of θ, this system allows for a robust receiver in the sense that the receiver does not have to re-adjust θ for different jamming environments or system parameters. Standard computation yields [10]

$$p_c(\theta) = 1 + \rho \sum_{k=1}^{M-1} \binom{M-1}{k} \frac{(-1)^k \theta}{k+\theta} \cdot \exp\left(-\frac{k}{k+\theta} \rho \frac{E_c}{N_J}\right) \quad (17)$$

$$p_e(\theta) = \frac{\rho}{M-1} \sum_{k=1}^{M-1} \binom{M-1}{k} \left(\frac{k}{k+\theta-1}\right) \frac{(-1)^{k+1}\theta}{k+\theta} \cdot \exp\left(-\frac{k+\theta-1}{k+\theta} \frac{E_c}{N_J}\right) \quad (18)$$

where $p_c(\theta)$ and $p_e(\theta)$ are the transition probabilities of the resulting M-ary symmetric errors and erasure channel with θ as a parameter shown in Fig. 3. The Viterbi decoder that ignores the erased positions and minimizes Hamming distance among the nonerased positions is the maximum likelihood decoder as long as the symbol error probability is less than $(M-1)/M$. Hence, the Chernoff bound reduces to the Bhattacharyya bound and the Chernoff parameter, which is now a function of θ is given by

$$D(\theta) = \max_{0<p\leq 1} [p_x(\theta) + 2\sqrt{p_e(\theta)\, p_c(\theta)}]^L \quad (19)$$

where $p_x(\theta) = 1 - p_e(\theta) - p_c(\theta)$.

For $M > 2$ with perfect interleaving and binary coding, the resulting channel is a binary errors and erasure channel with

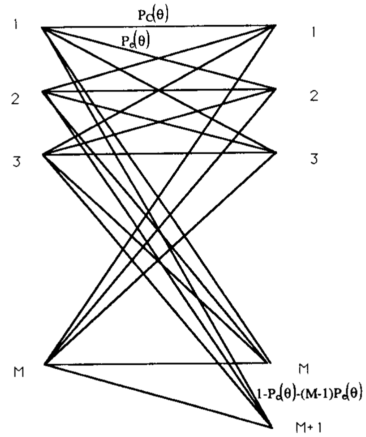

Fig. 3. M-ary errors and erasure symmetric channel.

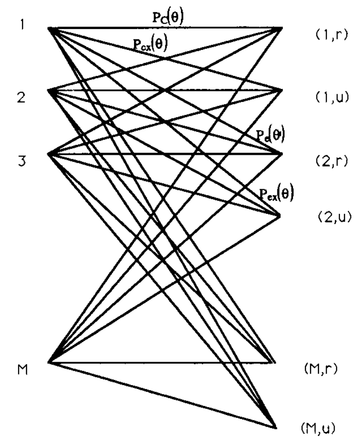

Fig. 4. M-ary to $2M$- ary symmetric channel.

correct probability $p_c^b(\theta)$, error probability $p_e^b(\theta)$ and erasure probability $p_x^b(\theta)$ given by

$$p_c^b(\theta) = p_c(\theta) + (M/2 - 1)p_e(\theta) \quad (20)$$

$$p_e^b(\theta) = (M/2)p_e(\theta) \quad (21)$$

$$p_x^b(\theta) = 1 - p_c^b(\theta) - p_e^b(\theta) \quad (22)$$

and $D(\theta)$ is again given by (19).

D. VRT Integer

In this system, Viterbi ratio thresholding is used to develop a quality bit q for the demodulated symbols. The rule is similar to the one described above except that when $1 \leq Y_{\max}/Y' < \theta$, we decide that the symbol corresponding to $Y_{\max}$ was sent and attach a quality bit q = 'u' to indicate that it is an *unreliable* decision and when $Y_{\max}/Y' \geq \theta$, we attach q = 'r' to indicate that it is a *reliable* decision. The resulting channel is an M-ary to $2M$-ary symmetric channel as shown in Fig. 4. The channel transition probabilities

$P_c(\theta), P_{cx}(\theta), P_{ex}(\theta), P_e(\theta)$ are given by

$$P_c(\theta) = p_c(\theta) \tag{23}$$

$$P_{cx}(\theta) = p_c(1) - p_c(\theta) \tag{24}$$

$$P_{ex}(\theta) = p_e(1) - p_e(\theta) \tag{25}$$

$$P_e(\theta) = p_e(\theta). \tag{26}$$

The metric $m((y,q),x)$ is assigned the following values where y is the detected symbol and q denotes the quality bit associated with y.

$$m((x,r),x) = N \tag{27}$$

$$m((y,r),x) = -N \tag{28}$$

$$m((x,u),x) = 1 \tag{29}$$

$$m((y,u),x) = -1 \tag{30}$$

where $x \neq y, x, y \in \{1, 2, \cdots M\}$ and N is an integer greater than equal to 1 that should be chosen appropriately. Straightforward analysis yields

$$D(\theta) = \max_{0<p\leq 1} \min_{\lambda\geq 0} [P_c(\theta)e^{-2\lambda N} + P_{cx}(\theta)e^{-2\lambda} + P_{ex}(\theta)e^{2\lambda} + P_e(\theta)e^{2\lambda N} + (M-2)P_e(\theta) + (M-2)P_{ex}(\theta)]^L. \tag{31}$$

For $M > 2$ and binary coding, the resulting coding channel is a binary to 4-ary symmetric channel with $P_c^b(\theta), P_{cx}^b(\theta), P_{ex}^b(\theta)$ and $P_e^b(\theta)$ given as follows [10]:

$$P_c^b(\theta) = p_c(\theta) + (M/2-1)p_e(\theta) \tag{32}$$

$$P_{cx}^b(\theta) = p_c(1) - p_e(\theta) + (M/2-1)(p_e(1) - p_e(\theta)) \tag{33}$$

$$P_{ex}^b(\theta) = (M/2)(p_e(1) - p_e(\theta)) \tag{34}$$

$$P_x^b(\theta) = (M/2)p_e(\theta). \tag{35}$$

Again $D(\theta)$ is given by (31) with $M = 2$.

E. Self-Normalized

Exact error probability analysis of this metric for BFSK and diversity was done in [3]. Here we resort to the Chernoff bound and compute D for $M \geq 2$. Again, let $Y_1, \cdots, Y_M$ be the outputs of the matched filters corresponding to the M modulated signals. Let us define ζ to be the sum of $Y_1, \cdots, Y_M$, i.e., $\zeta = \sum_{i=1}^{M} Y_i$. The self-normalized metric is defined as

$$m((Y_1 \cdots Y_M), l) = \frac{Y_l}{\zeta} \tag{36}$$

for $l \in \{1, 2, \cdots M\}$. Given that a 1 was transmitted and the particular hop was jammed, the probability distribution function (pdf) of the Y_i's are given by [4] and (37) below where $\gamma = \rho E_c/N_J$ and σ^2 is the jamming Gaussian noise power received during the chip interval. Also $I_0(\cdot)$ is the modified Bessel function of the first kind of order zero defined as [15]

$$I_0(z) = \frac{1}{\pi}\int_0^{\pi} e^{\pm z\cos(\theta)}\, d\theta. \tag{38}$$

Making the transformation of variables,

$$\Omega_i = \frac{Y_i}{\zeta}, \qquad i \in \{1, \cdots, M-1\} \tag{39}$$

$$\Omega_M = \zeta \tag{40}$$

and using the series representation $I_0(\sqrt{x}) = \sum_{n=0}^{\infty} (x/4)^n / n!n!$ [15], we obtain the joint pdf of $(\Omega_1, \cdots, \Omega_{M-1})$ as follows:

$$f_{\Omega_1,\cdots,\Omega_{M-1}}(\omega_1, \cdots, \omega_{M-1}) = e^{-\gamma}\sum_{n=0}^{\infty} \frac{(n+M-1)!}{n!n!}(\gamma\omega_1)^n \tag{41}$$

$$= (M-1)!\exp(x-\gamma)L_{M-1}^0(-x)|_{x=\gamma\omega_1} \tag{42}$$

for $0 \leq \sum_{i=1}^{M-1} \omega_i \leq 1$ and 0 otherwise. Here, $L_n^\alpha(\cdot)$ are the Laguerre polynomials [16] defined as

$$L_n^\alpha(x) = e^x \frac{x^{-\alpha}}{n!}\frac{d^n}{dx^n}(e^{-x}x^{n+\alpha}), \quad n = 0, 1, 2, \cdots \tag{43}$$

for any real $\alpha > -1$. Hence, if we define

$$g(x) = \frac{d^{M-1}}{dx^{M-1}}(e^x x^{M-1}), \tag{44}$$

we may write

$$f_{\Omega_1,\cdots,\Omega_{M-1}}(\omega_1, \cdots, \omega_{M-1}) = e^{-\gamma} g(\gamma\omega_1) \tag{45}$$

for $0 \leq \sum_{i=1}^{M-1} \omega_i \leq 1$. When the hop is not jammed, $\Omega_1 = 1$ and $\Omega_i = 0$ for $i = 2, \cdots, M-1$ with probability one.

For the case when $M = 2$, the Chernoff parameter $D(\rho, \lambda)$ is derived as follows:

$$D(\rho,\lambda) = [(1-\rho)e^{-\lambda} + \rho E\{\exp(\lambda(m((Y_1,Y_2),2) - m((Y_1,Y_2),1)))|1\}]^L \tag{46}$$

$$= [(1-\rho)e^{-\lambda} + \rho E\{\exp(\lambda(1-2\Omega_1))|1\}]^L \tag{47}$$

$$= \left[(1-\rho)e^{-\lambda} + \frac{\rho}{(2\lambda-\gamma)^2}(2\lambda e^{-\gamma+\lambda} + (\gamma^2 - 2\lambda - 2\lambda\gamma)e^{-\lambda})\right]^L \tag{48}$$

where the expectation is with respect to Ω_1 given that the hop is jammed and 1 was transmitted. Also, $f_{\Omega_1}(\omega_1) = (1+\gamma\omega_1)\exp(\gamma\omega_1 - \gamma)$ for $0 \leq \omega_1 \leq 1$ by (45).

$$p_{Y_i}(y_i) = \begin{cases} \frac{1}{2\sigma^2}\exp\left(-\frac{y_1}{2\sigma^2} - \gamma\right) I_0\left(\sqrt{\frac{2y_1\gamma}{\sigma^2}}\right) & i = 1 \\ \frac{1}{2\sigma^2}\exp\left(-\frac{y_1}{2\sigma^2}\right) & i = 2, 3, \cdots, M \end{cases} \tag{37}$$

When $M \geq 4$, we need to find the joint distribution of (Ω_1, Ω_2) in order to compute the Chernoff parameter. Integrating out the unwanted variables, the joint pdf of (Ω_1, Ω_2) is given by

$$\begin{aligned} f_{(\Omega_1,\Omega_2)}(\omega_1,\omega_2) &= \int_0^{1-\omega_1-\omega_2} d\omega_3 \cdots \int_0^{1-\omega_1-\cdots-\omega_{M-2}} \\ &\quad \cdot d\omega_{M-1} f_{\Omega_1,\cdots,\Omega_{M-1}}(\omega_1,\cdots,\omega_{M-1}) \\ &= e^{-\gamma} g(\gamma\omega_1) \int_0^{1-\omega_1-\omega_2} d\omega_3 \cdots \int_0^{1-\omega_1-\cdots-\omega_{M-2}} d\omega_{M-1}. \end{aligned} \tag{49}$$

Therefore, $D(\rho, \lambda)$ can be derived as follows:

$$\begin{aligned} D(\rho,\lambda) &= [(1-\rho)e^{-\lambda} + \rho E\{\exp(\lambda(m((Y_1,Y_2),2) \\ &\quad - m((Y_1,Y_2),1)))|1\}]^L \qquad (50) \\ &= [(1-\rho)e^{-\lambda} + \rho e^{-\gamma} \int_0^1 d\omega_1 e^{-\lambda\omega_1} g(\omega_1) h(\omega_1)]^L \qquad (51) \end{aligned}$$

where

$$\begin{aligned} h(\omega_1) &= \int_0^{1-\omega_1} d\omega_2 e^{\lambda\omega_2} \int_0^{1-\omega_1-\omega_2} \\ &\quad \cdot d\omega_3 \cdots \int_0^{1-\omega_1-\cdots-\omega_{M-2}} d\omega_{M-1}. \end{aligned} \tag{52}$$

The expression for $g(\cdot)$ and $h(\cdot)$ are easily computed from (44) and (52) with the help of a symbolic processing package. For the case when $M = 4$,

$$g(\gamma\omega_1) = e^x(x^3 + 9x^2 + 18x + 6)|_{x=\gamma\omega_1} \tag{53}$$

$$h(\omega_1) = \frac{1}{\lambda^2}(\lambda\omega_1 - \lambda - 1 + e^{\lambda(1-\omega_1)}). \tag{54}$$

For $M = 8$

$$\begin{aligned} g(\omega_1) &= e^x(x^7 + 49x^6 + 882x^5 + 7350x^4 \\ &\quad + 29400x^3 + 52920x^2 + 35280x + 5040)|_{x=\gamma\omega_1} \qquad (55) \end{aligned}$$

$$\begin{aligned} h(\omega_1) &= \left(\frac{1}{120\lambda^6}\right)\{\lambda^5(\omega_1^5 - 5\omega_1^4 + 10\omega_1^3 - 10\omega_1^2 + 5\omega_1 - 1) \\ &\quad + \lambda^4(-5\omega_1^4 + 20\omega_1^3 - 30\omega_1^2 + 20\omega_1 - 5) \\ &\quad + \lambda^3(20\omega_1^3 - 60\omega_1^2 + 60\omega_1 - 20) \\ &\quad + \lambda^2(-60\omega_1^2 + 120\omega_1 - 60) \\ &\quad + \lambda(120\omega_1 - 120) \qquad (56) \\ &\quad - 120 + 120e^{\lambda(1-\omega_1)}\}. \qquad (57) \end{aligned}$$

For $M \geq 16$, the expressions for $g(\cdot)$ and $h(\cdot)$ becomes quite complex.

For any jamming system where a hop is jammed in its entirety if it is jammed at all, asymptotically as $M \to \infty$, it is always possible to distinguish between hops that are jammed and the hops that are not jammed by observing the outputs of the matched filters.[1] In order to determine how large an M we need to obtain an accurate information as to which of the hops were jammed, we shall compute E_b/N_J needed to achieve computational cutoff rate (denoted by $E_b/N_J|_{R_0}$) for the self-normalized metric with and without side-information and compare this with $E_b/N_J|_{R_0}$ for the square-law-combining metric with perfect side-information.

[1] This is true even if the background noise is not negligible.

Following [4] we define the cutoff rate of a channel with M-ary input alphabet as follows:

$$R_0 = 1 - \log_M(1 + (M-1)D) \quad \text{[information symbols/channel use]}. \tag{58}$$

The computational cutoff rate is thought to be the *practically achievable* reliable data rate when coding is employed. We are interested in the bit signal-to-noise ratio needed to achieve cutoff rate given by

$$\left.\frac{E_b}{N_J}\right|_{R_0} = D^{-1}\left(\frac{M^{1-R_0} - 1}{M - 1}\right). \tag{59}$$

For the self-normalized metric with perfect side-information, $D(\rho, \lambda)$ is given by

$$D(\rho,\lambda) = \rho e^{-\gamma} \int_0^1 d\omega e^{-\lambda\omega} g(\omega)\, h(\omega). \tag{60}$$

F. Soft SI, Hard SI

Expressions of D for these metrics are given in [4] as

$$D = \max_{0<\rho\leq 1} \min_{\lambda \geq 0} \left[\frac{\rho}{1-\lambda^2} \exp\left(\frac{-\lambda}{\lambda+1}\left(\rho\frac{E_c}{N_J}\right)\right)\right]^L \tag{61}$$

for square-law-combining metric with perfect side-information and

$$D = \max_{0<\rho\leq 1} \left[\rho\left(\sqrt{\frac{4P_S(1-P_s)}{M-1}} + \left(\frac{M-2}{M-1}\right)P_S\right)\right]^L \tag{62}$$

for hard decisions and Hamming distance metric with perfect side-information, where

$$P_S = \sum_{l=1}^{M-1} \binom{M-1}{l} \frac{(-1)^{l+1}}{l+1} \exp\left(-\frac{l}{l+1}\rho\frac{E_c}{N_J}\right). \tag{63}$$

For hard decisions and Hamming distance metric with side-information and binary coding for MFSK ($M > 2$),

$$D = \max_{0<\rho\leq 1} [2\rho\sqrt{p(1-p)}]^L \tag{64}$$

where p is given by (10).

III. Numerical Results and Discussion

In this section, numerical results are given for the metrics considered in this paper for various performance measures. The codes we consider are rate 1/2, constraint length 7, binary convolutional codes and rate 1/2 dual-k codes concatenated with *optimal* diversity that minimizes the signal-to-noise ratio needed to guarantee the Union–Chernoff bound on the bit error rate to be less than 10^{-5}. For regions of interest, the Union–Chernoff bound on the bit error probability for the rate 1/2 optimal binary code can be closely approximated as $P_b \lesssim \frac{1}{2}(36D^{10} + 211D^{12} + 1404D^{14} + 11633D^{16})$ [13] and for dual-k codes, (7) is used to upper bound P_b.

For Viterbi ratio thresholding with integer metric, the performance of the system remained fairly constant (within 0.1

TABLE I
$E_b/N_J|_{10^{-5}}$, Rate [b/dimension], $L_{\rm opt}$, $\theta_{\rm opt}$

System Type	Coding	M	$E_b/N_J\|_{10^{-5}}$ dB	Rate [bits/dimension]	L_{opt}	θ_{opt}
Hard No SI	Binary	2	13.3	0.0833	3	
		4	11.4	0.0833	3	
		8	10.5	0.0625	3	
		16	10.2	0.0417	3	
		32	10.1	0.0260	3	
	Dual-k	4	13.0	0.0417	6	
		8	11.4	0.0375	5	
		16	10.5	0.0250	5	
		32	10.0	0.0156	5	
Hard E&X No SI	Binary	2	13.8	0.0625	$L_{opt}=4$, $T_{opt}=0$	
VRT E&X	Binary	2	12.3	0.1250	2	3.0
		4	10.4	0.1250	2	2.2
		8	9.5	0.0938	2	1.9
		16	8.9	0.0625	2	1.9
		32	8.6	0.0391	2	1.7
	Dual-k	4	12.6	0.0500	5	1.5
		8	11.3	0.0375	5	1.2
		16	10.5	0.0250	5	1.0
		32	10.0	0.0156	5	1.0
VRT Integer	Binary	2	11.9	0.1250	2	6.0
		4	9.7	0.1250	2	3.5
		8	8.7	0.0938	2	3.0
		16	8.2	0.0625	2	2.5
		32	7.9	0.0391	2	2.2
	Dual-k	4	11.9	0.0833	3	4.0
		8	10.6	0.0625	3	4.0
		16	9.9	0.0417	3	4.0
		32	9.5	0.0260	3	4.0
Self-Normalized	Binary	2	11.5	0.1250	2	
	Dual-k	4	10.9	0.0833	3	
		8	9.3	0.0625	3	
Soft SI	Binary	2	10.9	0.1250	2	
	Dual-k	4	10.7	0.0833	3	
		8	9.2	0.0625	3	
		16	8.2	0.0417	3	
		32	7.5	0.0260	3	
Hard SI	Binary	2	12.3	0.1250	2	
		4	10.1	0.1250	2	
		8	9.0	0.0938	2	
		16	8.5	0.0625	2	
		32	8.1	0.0931	2	
	Dual-k	4	12.1	0.0833	3	
		8	10.7	0.0625	3	
		16	9.9	0.0417	3	
		32	9.5	0.0260	3	

dB) for values of N between 3 and 6. We chose $N = 3$ which was the value used in [10].

Table I tabulates $E_b/N_J|_{10^{-5}}$ with optimal diversity. Also shown are the optimal rates in [bits/(signal) dimension] and optimal θ's for metrics using Viterbi ratio thresholding. It shows that the self-normalized metric is the best among the robust metrics considered in this paper. For $M = 8$ and dual-k coding, the self-normalized metric is within 0.1 dB from the square-law-combining metric with perfect side-information, 0.2 dB when $M = 4$ and 0.6 dB when BFSK is employed with binary convolutional coding. The gain over hard decisions without side-information is 2.1 dB for $M = 8$. Also, the optimal diversity level for the self-normalized metric is two less than that of hard decisions without side-information and is the same as the optimal diversity level of the square-law-combining metric with perfect side-information for all the codes considered.

With binary coding and $M > 2$, Viterbi ratio thresholding with integer metric seems to be a good choice, but with dual-k coding, it does not offer much gain over simple hard-decisions without side- information. The gain over simple hard decisions is only 0.5 dB for $M = 32$ with a dual-5 code where the gain is 2.2 dB with interleaved binary coding. The situation is even worse for Viterbi ratio thresholding with errors and erasure decision where the optimal θ actually reduces to 1.0 for dual-k coding as M increases, which implies that it is better not to use thresholding at all. As with Viterbi ratio thresholding with integer metric, this metric offers good performance with interleaved binary coding. This conforms with observations made in [10].

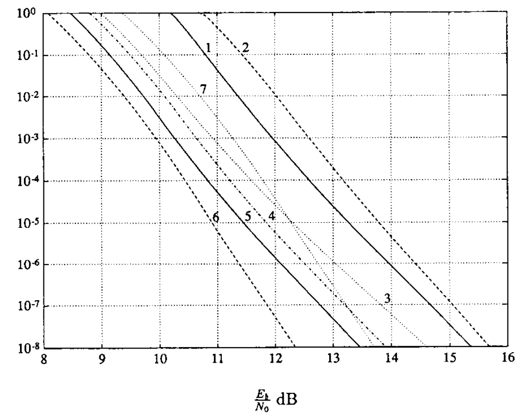

Fig. 5. P_b versus E_b/N_J, BFSK, binary code, optimal diversity.

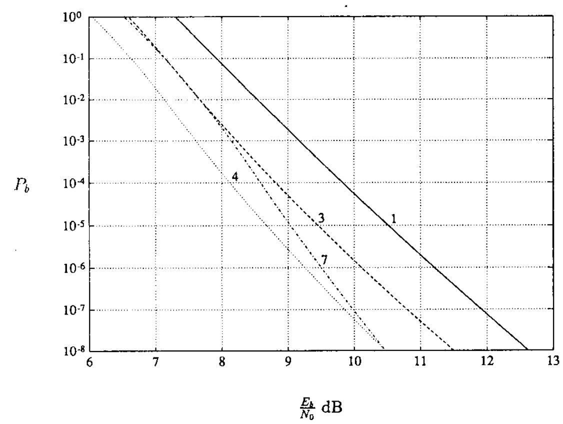

Fig. 6. P_b versus E_b/N_J, 8-FSK, binary code, optimal diversity.

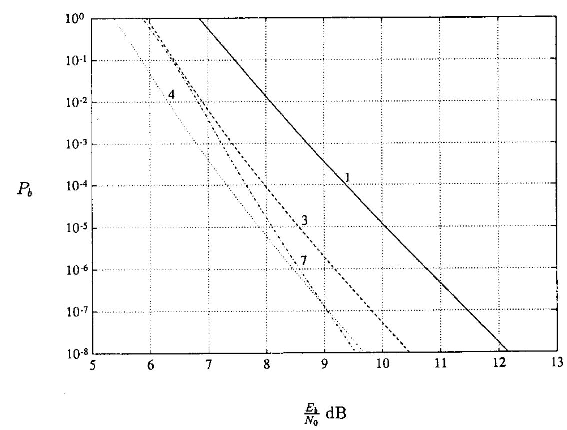

Fig. 7. P_b versus E_b/N_J, 32-FSK, binary code, optimal diversity.

Figs. 5–7 are the plots of the bound on the bit error probability versus the signal-to-noise ratio for $M = 2, 8, 32$ with binary coding and optimum integer diversity that minimizes the signal-to-noise ratio needed to achieve bit error rate of

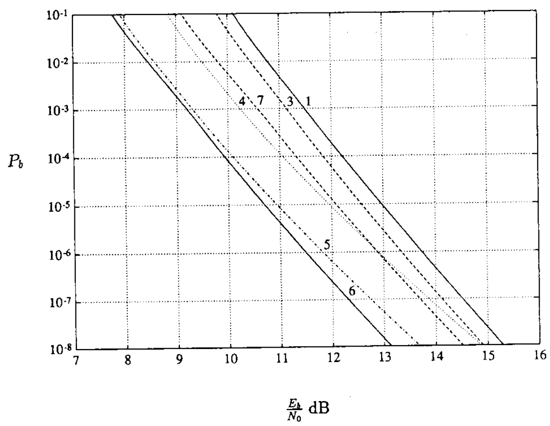

Fig. 8. P_b versus E_b/N_J, 4-FSK, dual-3 code, optimal diversity.

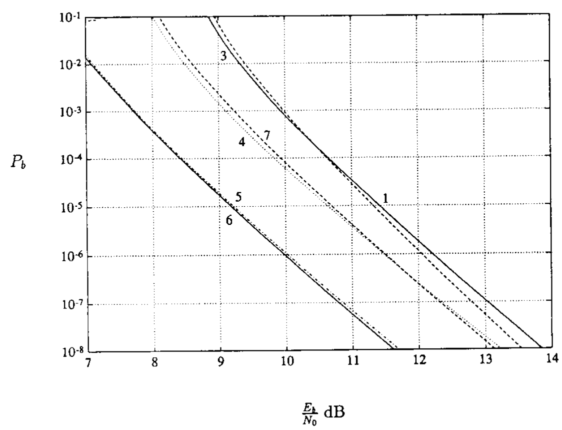

Fig. 9. P_b versus E_b/N_J, 8-FSK, dual-3 code, optimal diversity.

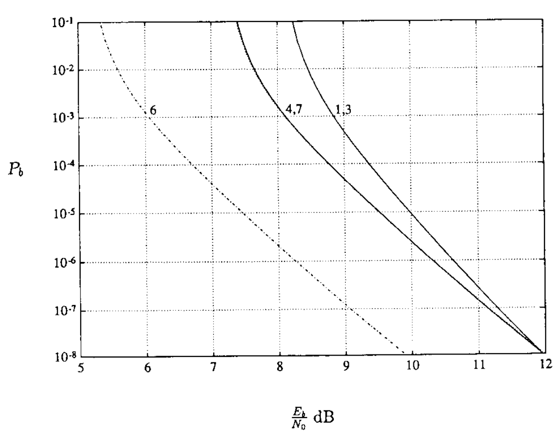

Fig. 10. P_b versus E_b/N_J, 32-FSK, dual-5 code, optimal diversity.

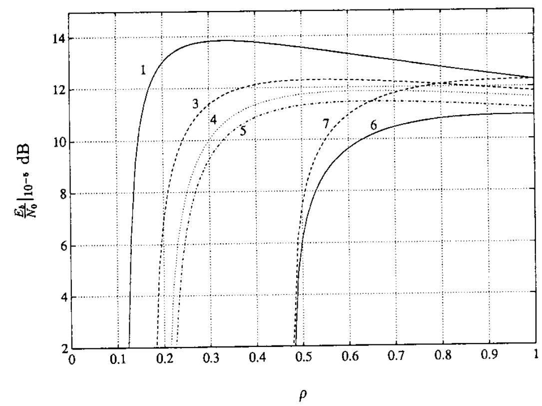

Fig. 11. $E_b/N_J|_{10^{-5}}$ versus ρ, BFSK, binary code, $L = 2$.

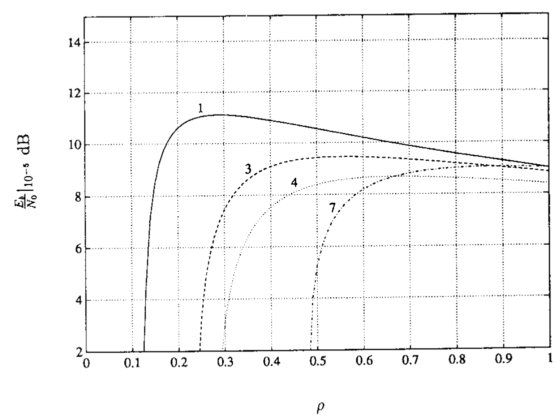

Fig. 12. $E_b/N_J|_{10^{-5}}$ versus ρ, 8-FSK, binary code, $L = 2$.

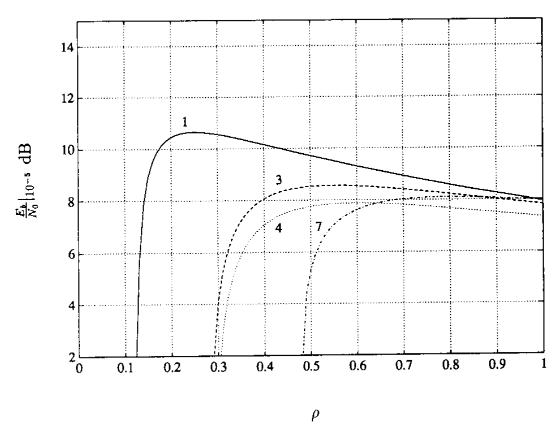

Fig. 13. $E_b/N_J|_{10^{-5}}$ versus ρ, 32-FSK, binary code, $L = 2$.

10^{-5}. Figs. 8–10 are similar plots for dual-k coding for $M = 4, 8$ and 32. We notice that for $M = 8$ and dual-3 coding, the self-normalized metric is practically indistinguishable from the square-law-combining metric with perfect side-information.

In Figs. 11–13, the plots of signal-to-noise ratios needed to achieve bit error rate of 10^{-5} versus ρ are given for $M = 2, 8, 32$ with binary coding and diversity level 2. Figs. 14–15 are similar plots for dual-k coding with diversity level 3 and $M = 8, 32$. Again Fig. 14 shows that the self-normalized metric has narrow-band noise rejection capabilities comparable

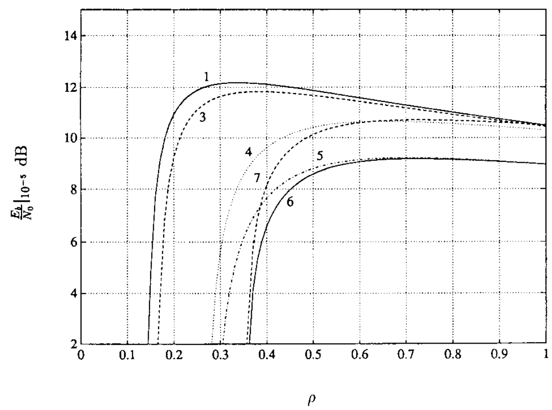

Fig. 14. $E_b/N_J|_{10^{-5}}$ versus ρ, 8-FSK, dual-3 code, $L = 3$.

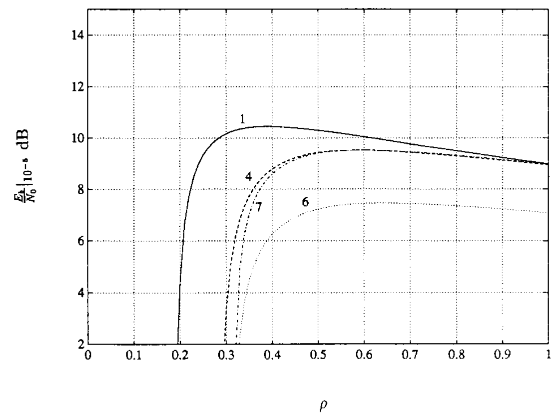

Fig. 15. $E_b/N_J|_{10^{-5}}$ versus ρ, 32-FSK, dual-5 code, $L = 3$.

to that of the square-law-combining metric with perfect side-information for $M = 8$.

Fig. 16 is the signal-to-noise ratio needed to achieve computational cutoff rate versus the cutoff rate for self-normalized metric, self-normalized metric with perfect side-information, square-law-combining metric with perfect side-information, hard decisions without side-information and hard decisions with perfect side-information for $M = 8$. This shows that the self-normalized metric effectively suppresses the contribution of the jammed hops on the decision process.

Finally, instead of concatenating diversity L with a rate 1/2 convolutional code, we may consider the performance of optimal convolutional codes with rate $r = 1/2L$ given in [14]. These two systems will have the same overall rate. We give the performance of some of the metrics with rate $1/2L$ binary and dual-k convolutional coding in Table II. We note that there is practically no difference (to the first decimal of $E_b/N_J|_{10^{-5}}$) between systems that use optimal convolutional codes and systems that concatenate diversity with rate 1/2 convolutional codes. This allows one to use, with negligible loss, diversity concatenated with rate 1/2 convolutional codes (the decoder for which is readily available) to achieve a desired effective code rate.

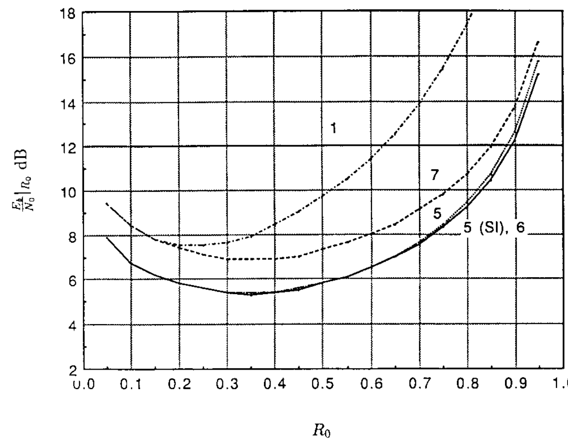

Fig. 16. $E_b/N_J|_{R_0}$ for hard decision metric with and without side- information, self-normalized metric, square-law-combining metric with perfect side-information. $M = 8$.

TABLE II
$E_b/N_J|_{10^{-5}}$. Rate [b/dimension], $r_{\mathrm{opt}}, \theta_{\mathrm{opt}}$

System Type	Coding	M	$E_b/N_J\|_{10^{-5}}$ dB	Rate [bits/dimension]	$r_{opt} = 1/(2L_{opt})$	θ_{opt}
VRT E&X.	Binary	2	12.3	0.1250	1/4	3.0
VRT Integer	Binary	2	11.9	0.1250	1/4	6.0
	Dual-k	8	10.6	0.0625	1/6	4.0
Self-Normalized	Binary	2	11.5	0.1250	1/4	
	Dual-k	8	9.3	0.0625	1/6	
Soft SI	Binary	2	10.9	0.1250	1/4	
	Dual-k	8	9.2	0.0625	1/6	
Hard SI	Binary	2	12.3	0.1250	1/4	
	Dual-k	8	10.7	0.0625	1/6	

IV. Conclusion

In this paper, we considered various robust metrics with diversity and error-correction coding for a partial-band noise jammed channel with FHSS modulation. We found that good robust soft metrics perform very close to square-law-combining metric with perfect side-information. The self-normalized metric performed especially well with performance almost identical to that of the square-law-combining metric with perfect side-information for $M = 8$. We considered the on-off partial-band noise jammer as the source of interference. It is known that [6,17] this two-level jammer is the worst case jammer for metrics (1), (6), and (7). It is also known [17] that for Gaussian noise jamming, multilevel jamming strategy does not significantly degrade performance of the communicator over the two level on-off jamming.

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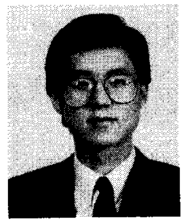
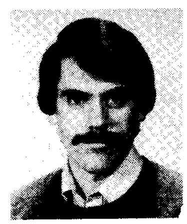


Kyungwhoon Cheun was born in Seoul, Korea, on December 16, 1962. He received the B.A. degree in electronic engineering from Seoul National University in 1985 and the M.S. and Ph.D. degrees from the University of Michigan, Ann Arbor, in 1987 and 1989, respectively, both in electrical engineering.

From 1987 to 1989, he was a Research Assistant at the Department of Electrical Engineering and Computer Science at the University of Michigan and from 1989 to 1991 he was an Assistant Professor at the University of Delaware. In June of 1991, he joined the Department of Electrical Engineering at the Pohang Institute of Science and Technology, Pohang, Korea, where he is currently and Assistant Professor. His current research interests include Spread-spectrum systems for antijam and multiple-access systems, synchronization and equalization techniques for radio communication systems and the application of error correcting codes.



Wayne E. Stark (S'77–M'82) was born on February 26, 1956. He received the B.S. (with highest honors), M.S., and Ph.D. degrees in electrical engineering from the University of Illinois, Urbana-Champign, in 1978, 1979, and 1982, respectively.

Since 1982 he has been a faculty member at the University of Michigan where he is currently an Associate Professor of Electrical Engineering and Computer Science. His research interests include information theory, coding theory, communication theory, especially for spread-spectrum communication systems.

Dr. Stark received the 1985 National Science Foundation's Presidential Young Investigator Award; was a member of the Board of Governors of the IEEE Information Theory Group and was an Editor for the IEEE TRANSACTIONS ON COMMUNICATIONS from 1984 to 1989. He participated in the organization of the 1986 IEEE International Symposium on Information Theory, Ann Arbor, MI. In 1989 he was on sabbatical at IBM Zurich Research Laboratory, Rushlikon, Switzerland.