Optimum Rate Reed–Solomon Codes for Frequency-Hopped Spread-Spectrum Multiple-Access Communication Systems

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Abstract—**In this paper, we consider a multiple-access frequency-hopped spread-spectrum communication system with Reed–Solomon codes. The performance measures of interest are an achievable region and the channel throughput. The achievable rate region is the set of all pairs of code rate and number of users for which communication is possible with error probability below a fixed value. The throughput measures the expected number of successful codeword transmissions per unit bandwidth. Two models of interference are considered. For these two models, we determine the optimal number of users for a given bandwidth and the optimal rate Reed–Solomon code that maximize the throughput. We also determine the achievable region for these models.**

I. Introduction

In this paper, we consider a frequency-hopped multiple-access spread-spectrum communication system utilizing Reed–Solomon codes. In a frequency-hopped multiple-access system, errors occur primarily due to multiple-access interference. We will consider two models for the demodulator. In the first model, the channel for each user is a simple M-ary symmetric channel with the error probability dependent on the number of users transmitting. Our second model assumes it is possible to determine which of the transmitted symbols were subject to multiple-access interference and which were not. If this is possible, a symbol that is interfered with is erased and a decoding error will occur only if the number of erasures is greater than the erasure correcting capability of the code. Since the erasure correcting capability of a code is twice the error correcting capability, we expect that erasure correction will perform significantly better than error correction.

In a multiple-access system many users are desired to share a given bandwidth with a given error probability. We consider a packetized transmission scheme. Time is slotted into intervals of length equal to the time it takes to transmit a packet (plus a guard time). Each packet is taken to be a single codeword. One performance measure of interest is the throughput of the multiple-access channel which is the average number of successful packet transmissions per unit time. One way of possibly increasing the total throughput is to increase the number of users simultaneously transmitting. However, as the number of users increases, the probability of error increases (because users will be more likely to hop to the same frequency slot). Thus, we expect there to be an optimal number of users that should simultaneously transmit to maximize the throughput of reliable information. If we also normalize the throughput by the code rate being used to account for the greater bandwidth needed by coding, then there is also an optimal rate code that should be used. Another performance measure that one may be interested in is the set of pairs of rate and number of users for which communication is possible with packet error probability below a certain desired value. We call this the achievable region. This performance measure allows one to determine, for any code rate, the maximum number of users that can simultaneously transmit, or for any given number of users, the maximum code rate that can be used for the probability of success to be above a desired value.

In this paper, we optimize the normalized throughput over the number of users and the code rate and then jointly optimize over both the number of users and code rate. As an example of the results we obtain, we show that for Reed–Solomon codes with perfect information about "hits" from other users during a given symbol, the code rate that maximizes the throughput is e^{-1} for very long Reed–Solomon codes with the optimal number of users. We also show that this turns out to be very accurate for practical length codes (e.g., length 32). Without side information, we show that the optimal code rate (again for the optimal number of users) is roughly 0.45 depending slightly on the specific nature of errors given at least one hit from another user occurs. We also derive the region or set of code rates and number of users for which communication is possible at a given error rate (for each user).

Similar work on throughput for frequency-hopped systems has been presented in [1] and [2]. Our goal in this paper (besides computing the achievable region) is to examine the optimal code rate and number of users for finite length codes and finite number of users to maximize the throughput. To this end, we have derived approximations to the throughput which give very accurate approximations to the optimal code rate and optimal number of users.

II. Channel Model and Performance Measures

In this section, we briefly describe the channel model for the frequency-hopped spread-spectrum multiple-access communication system we are considering. Since there are many expositions of the channel model given in other papers (see [3], [4]), we do not go into the detailed waveform channel. We also define the performance measures of interest.

The multiple-access problem we address in this paper is a problem with K transmitter-receiver pairs. Each transmitter-

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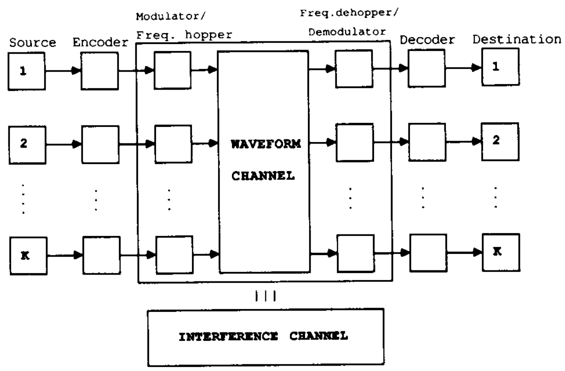

Fig. 1. K user frequency-hopped spread-spectrum multiple-access channel.

receiver pair has a unique hopping pattern. Since, in general, it is unlikely that the system designer will be able to incorporate the different hopping patterns in designing an optimal system, we model the hopping patterns as independent, identically distributed (i.i.d.) random sequences uniformly distributed over a set of q frequencies. The basic channel block diagram is shown in Fig. 1. The data (M-ary) from each source are encoded by identical Reed–Solomon encoders. Reed–Solomon codes are used because of their good burst error-correcting capability. In fact, we will show that they achieve capacity for one of the channels considered when the block length becomes very long. We will consider both binary and M-ary frequency-shift keyed forms of modulation. Each of these signals is frequency-hopped over a set of q frequencies. The input to the receiver then consists of the sum of the K transmitters' output plus background noise (which we ignore for simplicity). As one possibility we consider the situation in which the demodulator can determine if any other user's frequency-hopped signal used the same frequency as that used by the corresponding frequency-hopper. We call this the case of (perfect) side information available. The decoder then processes the received symbols (which may contain erasures).

We will consider both the asynchronous hopping case in which hopping sequences for different users do not necessarily change at the same time, and the synchronous case. The former is the more realistic situation. Given that there are q frequency slots available and that all hopping patterns are independent it is well known [3], [4] that the probability $P_{h,K}$ of at least one of the $K - 1$ other users hopping to the same frequency as a given user during that user's dwell time (i.e., for some or all of the dwell time) is given by

$$P_{h,K} = 1 - (1 - p_h)^{K-1}$$

where

$$p_h = \begin{cases} \dfrac{2}{q} - \dfrac{1}{q^2}, & \text{asynchronous} \\ \dfrac{1}{q}, & \text{synchronous} \end{cases}$$

is the probability of another user hopping (for some portion of the dwell time) to the same frequency. In this expression, and throughout the paper, we assume that one symbol is transmitted per hop interval.

The coding channel for each user is then a discrete memoryless channel. The input alphabet is the set $\{0, 1, \cdots, M-1\}$. The output alphabet is either $\{0, 1, \cdots, M-1\}$ or $\{0, 1, \cdots, M-1, ?\}$ where the symbol ? denotes an erasure. The latter alphabet is used when the receiver has information about the interference present during each hop and can erase the received symbol corresponding to that hop. The models we will consider will be such that the output of the channel for a given user does not depend on the input to the channel for other users. This is the so-called "separated" case for interference channels [5]. (See [6] for a model and analysis of the nonseparated case.) In the separated case, the channel can be decomposed into K single-user channels. To complete the description of the channel, we need to determine the transition probabilities from the input of the channel to the output of the channel. For the case of perfect side information available, the transition probabilities are given by

$$p(y|x) = \begin{cases} 1 - P_{h,K}, & y = x \\ 0, & y \neq x, y \neq ? \\ P_{h,K}, & y = ? \end{cases}$$

where y is the channel output and x is the channel input. For the case when side information is unavailable, the channel is modeled as a M-ary symmetric discrete memoryless channel with error probability $P_{h,K}(M-1)/M$. This corresponds to a model in which if a hit occurs, the demodulator is equally likely to choose any of the transmitted symbols. The transition probabilities are then

$$p(y|x) = \begin{cases} 1 - \left(P_{h,K} \dfrac{(M-1)}{M}\right), & y = x \\ \dfrac{(M-1)}{M} P_{h,K}, & y \neq x. \end{cases}$$

While this model may seem overly pessimistic, the optimal code rate found is quite close to the code rate determined in [6] for the more realistic model. The more realistic model considers an M-ary FSK modulator. If there is just one hit, the channel has error probability $M/2(M-1)$, i.e., 1/2 times the probability that the users do not transmit the same symbol. If there are two hits, the error probability is determined as 1/2 times the probability that the two interfering users transmit the same symbol which is different than the symbol of the user of interest plus 2/3 times the probability that the two users transmit different symbols that also differ from the symbol of the user of interest. Because the results obtained for this more complicated model are in the same spirit as the results for the simpler model, we only consider the simpler case in this paper. In general, the simplified model is pessimistic with respect to the more realistic model. Furthermore, as will be shown in a subsequent paper, when concatenated codes are employed, the pessimistic model performs identical to the more realistic model and performs as well as if perfect side information were available.

In the next section, we will use these transition probabilities to determine the throughput of the different channels. The throughput is a measure of the average number of successful transmissions per unit time. The throughput is defined as (see [7])

$$s(K, n, k, q) = K(1 - P_e(K, n, k, q)) \tag{1a}$$

where $P_e(K, n, k, q)$ is the error probability at the output of the decoder for a given user. Also n and k are the length and dimension of the Reed-Solomon code being used by each transmitter. We have explicitly displayed the dependence on K, n, k and q because we will later let $n, k \to \infty$ such that $k/n \to r$ and optimize over K and r and then let $q \to \infty$. For fair comparison to other nonspread systems or uncoded systems, we define the normalized throughput as

$$w(K, n, k, q) = s(K, n, k, q)r/q \tag{1b}$$

where $r = k/n$, $0 \leq r \leq 1$ is the rate of code being used by each user and q is the number of hopping frequencies.

In this paper, we assume that the number of transmitter-receiver pairs K is fixed for all time. We could also assume that K is a random variable if all users begin transmission of a codeword at the same time, i.e., that the number of interfering users is a constant during each transmission of the codeword. Under this condition the throughput is the average of the throughput determined here, averaged over the number of users.

From the above definition of throughput it is not clear which value of K is desirable. On the one hand, as K increases the amount of information being transmitted over the q frequency slots is increasing, but at the same time the reliability of that information is decreasing. Similarly, for the normalized throughput, as the code rate gets smaller the probability of successful transmission, and hence $s(K)$ is increasing, offsetting this is the normalization by the code rate. Thus, we would expect there to be an optimal code rate that maximizes the normalized throughput.

One more performance measure of interest is the achievable region. This region is the set of all vectors, the first component being the number of users, the second component being the rate of the code, such that the error probability is less than a given desired value. This region is useful in designing a system and for systems that adapt the code rate to the channel traffic, i.e., use a larger code rate for small channel traffic and a low rate for high channel traffic. Because of the difficulty in computing the achievable region, we describe an approximation that is very accurate for most values of number of users and code rates of interest.

III. Achievable Region and Throughput with Side Information

As mentioned previously, we will consider Reed–Solomon codes of length n and dimension k. The code rate is then k/n. The error and erasure correcting capability of Reed–Solomon codes are well known (see [8]). An (n, k) Reed–Solomon code can correct up to $\lfloor (n-k)/2 \rfloor$ errors and up to $n-k$ erasures. Thus, on an M-ary erasure memoryless[1] channel the probability of codeword error is given by

$$P_e(K, n, k, q) = \sum_{i=n-k+1}^{n} \binom{n}{i} P_{h,K}^{i}(1-P_{h,K})^{n-i}. \quad (2)$$

Asymptotically, as n and k approach infinity while the code rate, $r = k/n$ is held constant, it can be shown that

$$P_e(K, r, q) \triangleq \lim_{\substack{n,k\to\infty \\ k/n \to r}} P_e(K, n, k, q) = \begin{cases} 0, & 1-r > P_{h,K} \\ 1/2, & 1-r = P_{h,K}. \\ 1, & 1-r < P_{h,K} \end{cases} \quad (3)$$

This implies that error free communication is possible with Reed–Solomon codes provided

$$r < (1-p_h)^{K-1} \quad (4a)$$

or equivalently

$$K < 1 + \frac{\ln(r)}{\ln(1-p_h)}. \quad (4b)$$

If we also let K and q become large such that $K/q \to \lambda$ then we obtain the region

$$\lambda < \frac{1}{\eta} \ln\left(\frac{1}{r}\right) \quad (5a)$$

or equivalently

$$r < e^{-\eta\lambda} \quad (5b)$$

where $\eta = 1$ for synchronous hopping and $\eta = 2$ for asynchronous hopping.

For finite block lengths, computing the region $A(\hat{P}_e) \triangleq \{(K, r) | P_e(K) < \hat{P}_e\}$ is a time consuming problem (especially for large n) of calculating $P_e(K, n, k, q)$ for each value of $k = 1, 2, \cdots, n$ and determining if $P_e(K, n, k, q) < \hat{P}_e$ is satisfied. We now present a very accurate approximation to the achievable region for block lengths of interest (e.g., $n = 32$). The idea is to approximate (2) by approximating the distribution of the number of erasures by a Gaussian random variable with identical mean and variance. This leads to

$$P_e(K, n, k, q) \approx 1 - \Phi\left(\frac{n-k-nP_{h,K}}{\sqrt{nP_{h,K}(1-P_{h,K})}}\right) \quad (6)$$

where $\Phi(x) = \int_{-\infty}^{x} (1/\sqrt{2\pi}) e^{-t^2/2}\, dt$ is the standard Gaussian distribution function. If we are interested in error probability $\hat{P}_e$ and if $\alpha = \Phi^{-1}(1 - \hat{P}_e)$ then we approximate $A(\hat{P}_e)$ by

$$r < 1 - P_{h,K} - \alpha\sqrt{P_{h,K}(1-P_{h,K})/n} \quad (7a)$$

or equivalently

$$K < 1 + \frac{\ln(\hat{r})}{\ln(1-p_h)} \quad (7b)$$

where

$$\hat{r} \triangleq \frac{r + \beta^2 + \beta\sqrt{2r(1-r) + \beta^2}}{1 + 2\beta^2} \quad (8)$$

and where $\beta = \alpha/\sqrt{2n}$. Notice that if $\hat{P}_e$ is fixed at any (nonzero) value and n becomes large while r remains constant, then the approximation converges to (4). This approximation to the achievable region $A(\hat{P}_e)$ is considerably simpler to compute than evaluating (2) for each value of r.

The asymptotic normalized throughput $w(K, n, k, q)$ can be calculated using (1b) and (3) as

$$w(K, r, q) \triangleq \lim_{\substack{n,k\to\infty \\ k/n \to r}} w(K, n, k, q) = \begin{cases} \frac{rK}{q}, & K < 1 + \frac{\ln(r)}{\ln(1-p_h)} \\ \frac{rK}{2q}, & K = 1 + \frac{\ln(r)}{\ln(1-p_h)}. \\ 0, & K > 1 + \frac{\ln(r)}{\ln(1-p_h)} \end{cases} \quad (9)$$

It is clear now that the value of K that maximizes $w(K, r, q)$ is

$$K^*(r, q) = 1 + \frac{\ln(r)}{\ln(1-p_h)}. \quad (10)$$

Actually, the value of K that maximizes $w(r, K)$ is just

[1] Actually, the channel is not memoryless but is very well approximated by a memoryless channel (see [9]).

slightly less than that given above. This is because we want to choose K as large as possible but less than $1 + \ln(r)/\ln(1 - p_h)$. Similar comments apply to the throughput calculated below and to the case when side information is not present. We will ignore this small difference in the remainder of the paper. For the value of K given in (10) the (normalized) throughput as a function of the rate of the code is

$$w^*(r, q) = \frac{r}{q}\left[1 + \frac{\ln(r)}{\ln(1 - p_h)}\right]. \tag{11}$$

If we further optimize this over the code rate we find the optimal code rate to be

$$r^*(q) = e^{-1}(1 - p_h)^{-1}. \tag{12}$$

The corresponding value of K is

$$K^*(q) = [\ln(1 - p_h)^{-1}]^{-1}. \tag{13}$$

The maximum throughput is then

$$w^*(q) = \frac{(qe)^{-1}}{(1 - p_h)\ln(1 - p_h)^{-1}}. \tag{14}$$

If we let q become large then the optimum number of users normalized by q, which we denote by λ^* is

$$\lambda^* = \lim_{q \to \infty} K^*(q)/q = \eta^{-1}, \tag{15}$$

and the optimum code rate is

$$r^* = \lim_{q \to \infty} r^*(q) = e^{-1}. \tag{16}$$

The normalized throughput for large q is

$$w^* = \lim_{q \to \infty} w^*(q) = e^{-1}/\eta. \tag{17}$$

We can also find the optimum code rate for a fixed number of users by maximizing (9) as

$$r^*(K, q) = (1 - p_h)^{K-1}. \tag{18}$$

With this value of r, the throughput as a function of the number of users is given by

$$w^*(K, q) = \frac{K(1 - p_h)^{K-1}}{q}. \tag{19}$$

Optimizing this over K now gives the same value as in (12) and (14). Thus, the optimization of throughput over r and K is invariant to the order of optimization.

In Fig. 2, the achievable regions for length 32 and 256 Reed–Solomon codes are shown. We also show the achievable region for very long Reed–Solomon codes and the approximations to the achievable region derived above. As can be seen, these approximations are very accurate even for length 32 codes. In Fig. 3, the (normalized) throughput for length 32 codes is shown and in Fig. 4 the (normalized) throughput for length 256 codes is shown. In each of these figures, we also show, for comparison purposes, the asymptotic throughput for long codes. From these figures we observe the following. The asymptotic optimal code rate e^{-1} gives a very good approximation to the optimal rate even for codes of length 32. For example, for length 32 codes the optimal code is the (32, 12) code with rate 0.375; whereas, the code rate predicted by (16) is 0.368 or has 11.77 information symbols. For length 256 codes the optimal code is the (256, 94) code with rate 0.367. The optimal number of users $K^*(r, q)$ predicted by (10) is also a reasonably good approximation. For example, the number of users predicted by (10) for the (32, 20) code is 24; whereas, the actual number is 20. Similarly, the optimal number of

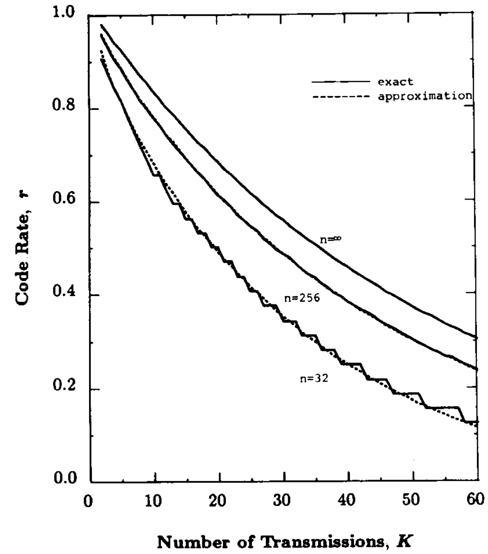

Fig. 2. Achievable region for $P_e \leq 10^{-2}$. (Perfect side information, $q = 100$.)

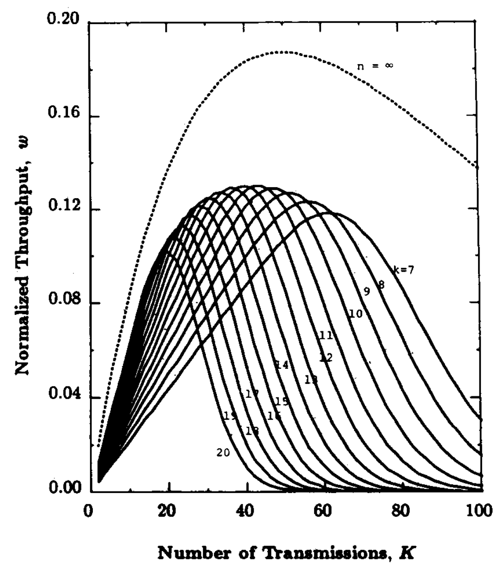

Fig. 3. Normalized throughput with perfect side information, $q = 100$, (32, k) Reed–Solomon codes, and $\eta = 2$.

users predicted by (10) is 38 for the (256, 120) code whereas the actual optimal number of users is 32.

IV. Achievable Region and Throughput Without Side Information

Without the availability of side information concerning multiple users occupying the same frequency slot at the same time the modeling of the interference becomes much more difficult. In this paper, we consider a very simple model which represents the worst case performance. We will assume that whenever two or more users occupy the same frequency slot at the same time, the probability of error is $(M - 1)/M$; that is, the demodulator is equally likely to choose any of the M different symbols. In [6], more realistic models for the error probability given a collision are analyzed. The results presented here will be lower bounds (on the throughput and achievable region) to any other model.

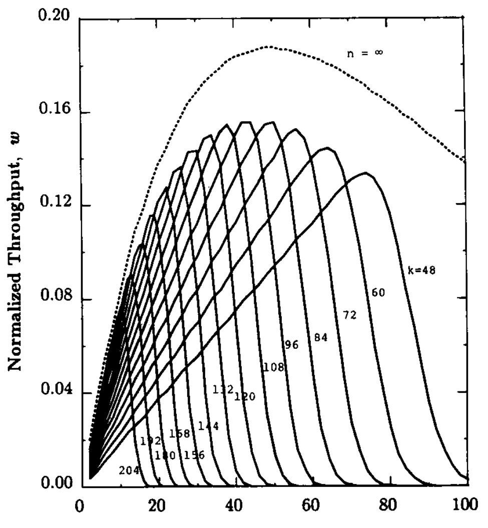

Fig. 4. Normalized throughput with perfect side information, $q = 100$, (256, k) Reed-Solomon codes, and $\eta = 2$.

The unconditional probability of error when side information is not available is given by

$$P_{e,K}=\frac{(M-1)}{M}[1-(1-p_h)^{K-1}].$$

With Reed-Solomon coding the error probability at the output of the decoder is given by

$$P_e(K, n, k, q)=1-\sum_{i=0}^{\lfloor (n-k)/2 \rfloor}\binom{n}{i}P_{e,K}^i(1-P_{e,K})^{n-i}.$$

Asymptotically, as n and k become large while $k/n \rightarrow r$ the decoder probability is given by

$$P_e(K, r, q)=\lim_{\substack{n,k\rightarrow\infty\\ k/n\rightarrow r}}=\begin{cases}0, & 1-r>2P_{e,K}\\ 0.5, & 1-r=2P_{e,K}.\\ 1, & 1-r<2P_{e,K}\end{cases}$$

This implies that error free communication is possible provided that

$$r<2(1-p_h)^{K-1}-1 \tag{20a}$$

or equivalently

$$K<1+\frac{\ln((1+r)/2)}{\ln(1-p_h)}. \tag{20b}$$

In arriving at (20), we have made use of the fact that for M-ary modulation with Reed-Solomon coding, each code symbol comes from an M-ary alphabet and that the length n of the Reed-Solomon code must be either M, $M-1$, or $M+1$. Thus, when n becomes large so does M.

If we also let K and q become large such that $K/q \rightarrow \lambda$ then we obtain the region

$$r<2e^{-\eta\lambda}-1$$

or equivalently

$$\lambda<\frac{1}{\eta}\ln\left(\frac{2}{1+r}\right).$$

For the case of finite length codes, we can derive a similar approximation to the achievable region as in the case with side information. We approximate the number of errors in a codeword of length n by a Gaussian random variable with the same mean and variance. With this the probability of error is approximated by

$$P_e(K, n, k, q)\approx 1-\Phi\left(\frac{\frac{(n-k)}{2}-nP_{e,K}}{\sqrt{nP_{e,K}(1-P_{e,K})}}\right) \tag{21}$$

where we have assumed that $n-k$ is even for simplicity. Using the above approximation, we obtain an approximation to the achievable region as

$$r<1-2P_{e,K}-2\alpha\sqrt{P_{e,K}(1-P_{e,K})/n} \tag{22a}$$

or equivalently

$$K\leq 1+\frac{\ln\left(\frac{1+\hat{r}}{2}\right)}{\ln(1-p_h)} \tag{22b}$$

where (after tedious algebra)

$$\hat{r}=\frac{\frac{M}{M-1}r-\frac{1}{M-1}(1+2\beta^2)+\frac{M\beta}{M-1}\sqrt{2(1-r^2)+4\beta^2}}{1+2\beta^2}, \tag{23}$$

$\beta = \alpha/\sqrt{2n}$, and $\alpha = \Phi^{-1}(1-\hat{P}_e)$. For fixed error probability $\hat{P}_e$ and for large n and M, $\hat{r}$ approaches r so that (22) approaches (20).

We now turn our attention to the normalized throughput. The normalized throughput for large n and k is

$$W(K, r, q) \triangleq \lim_{\substack{n,k\rightarrow\infty\\ k/n\rightarrow r}} W(K, n, k, q)$$

$$=\begin{cases} rK/q; & K<1+\frac{\ln((1+r)/2)}{\ln(1-p_h)}\\ rK/2q; & K=1+\frac{\ln((1+r)/2)}{\ln(1-p_h)}.\\ 0; & K>1+\frac{\ln(1+r)/2}{\ln(1-p_h)}\end{cases} \tag{24}$$

The throughput can be maximized over the number of users to yield

$$W^*(r, q)=\frac{r}{q}\left(1+\frac{\ln((1+r)/2)}{\ln(1-p_h)}\right).$$

The optimal number of users given by

$$K^*(r, q)=1+\frac{\ln((1+r)/2)}{\ln(1-p_h)}.$$

Maximization of the throughput with respect to the code rate is a nontrivial problem best solved numerically for each value of q of interest. The optimal value $r^*(q)$ along with $w^*(q)$ for many different values of q are given in Table I for asynchron-

TABLE I
OPTIMUM CODE RATE AND NUMBER OF USERS FOR MAXIMUM THROUGHPUT

q	$r^*(q)$	$K^*(q)$	$w^*(q)$
10.	0.659	1.9	0.1244
20.	0.552	3.5	0.0958
30.	0.520	5.0	0.0875
40.	0.504	6.6	0.0835
50.	0.495	8.2	0.0812
60.	0.489	9.8	0.0797
70.	0.485	11.3	0.0786
80.	0.482	12.9	0.0778
90.	0.479	14.4	0.0772
100.	0.477	16.1	0.0767
200.	0.468	31.8	0.0745
300.	0.465	47.6	0.0738
400.	0.464	63.3	0.0734
500.	0.463	79.1	0.0732
600.	0.463	94.8	0.0731
700.	0.462	110.6	0.0730
800.	0.462	126.3	0.0729
900.	0.462	142.0	0.0729
1000.	0.461	157.8	0.0728
2000.	0.461	315.3	0.0726
3000.	0.460	472.7	0.0725
4000.	0.460	630.2	0.0725
5000.	0.460	787.6	0.0725
6000.	0.460	945.1	0.0725
7000.	0.460	1102.6	0.0724
8000.	0.460	1260.0	0.0724
9000.	0.460	1417.5	0.0724
10000.	0.460	1574.9	0.0724

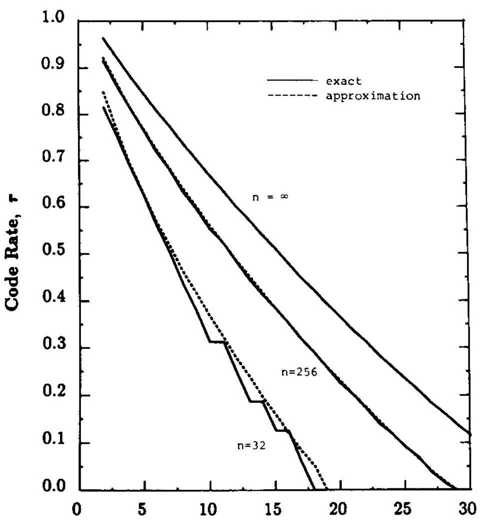

Fig. 5. Achievable region for $P_e \leq 10^{-2}$. (No side information, $q = 100$.)

ous systems. For large q the optimal code rate is $r^* = 0.4597$ and the corresponding throughput is $0.1448/\eta$. Also for large q, the number of users which maximize the throughput is given by

$$\lambda^* = \lim_{q \to \infty} K^*(q)/q = 0.3148/\eta.$$

We can also derive the optimal code rate as a function of the number of users to maximize throughput as

$$w^*(K, q) = (2(1-p_h)^{K-1} - 1)K/q$$

where

$$r^*(K, q) = 2(1-p_h)^{K-1} - 1.$$

Again, it is difficult to determine the optimal number of users as a function of q analytically. However, numerical computation is fairly easy. The optimal number of users K^* are also shown in Table I. The asymptotic results are the same as above for $q \to \infty$.

In Fig. 5 the achievable region for length 32 and 64 Reed-Solomon codes with no side information and word error probability of 10^{-2} is shown along with the achievable region for very long codes. In Fig. 6, the throughput is shown for the case $n = 32$ and $\eta = 2$ (asynchronous). As in the case with side information, the optimal code rate for long codes and a large number of hopping frequencies provides an excellent approximation for the codes of practical length. In Fig. 7, similar results are shown for the case $n = 256$.

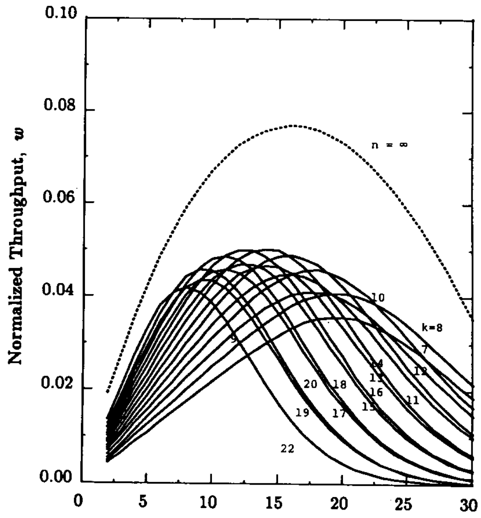

Fig. 6. Normalized throughput without side information, $q = 100$, $(32, k)$ Reed-Solomon codes, and $\eta = 2$.

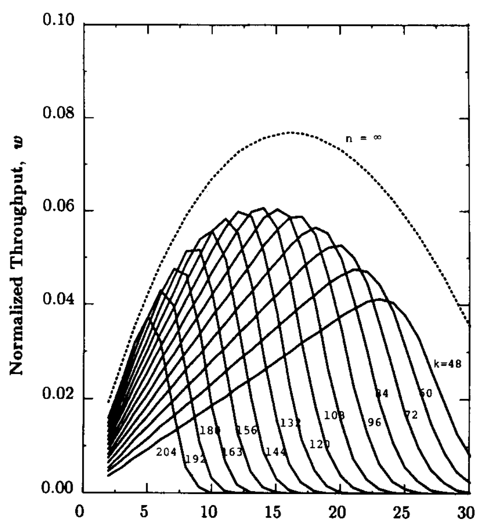

Fig. 7. Normalized throughput without side information, $q = 100$, $(256, k)$ Reed-Solomon codes, and $\eta = 2$.

V. CONCLUSIONS AND FURTHER DISCUSSION

In this paper, we have investigated the multiple-access capability of a frequency-hopped spread-spectrum system with Reed-Solomon coding. The effect of using side information at the receiver concerning multiple hops to the same frequency slot has been presented and compared to the case when side information is unavailable. We have determined in each case whether a given code rate and number of users is capable of having a certain error rate. We have also determined the optimum code rate and number of users to maximize the throughput. In this paper, we have only considered the case where the number of users is a constant. An extended analysis for the case where the number of users is distributed according to a Poisson distribution is given in [6]. Very similar results are obtained in that case. Furthermore, a more realistic (but more complicated) model for the case of no side information is considered.

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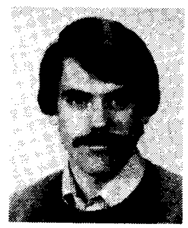

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